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Literature review

An integrated energy system on a national scale by 2050

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Abstract

Renewable energies play an important role in the transition to a low-carbon energy system in Europe, and provide an increasing part of European energy. However, the fluctuations in renewable energy production pose challenges in network management. Power-to-gas technology can be part of the solution by storing excess electricity on the short and long term. In this report, we study the different models for power network and gas network modelling, as well as the ways these networks are combined in the literature. Particular attention is paid to linear models. We take a brief look at software and example networks. Overall, the same few models are used throughout the literature, with variations depending on the objectives of each paper.

Nomenclature

Abbreviations

DC	direct current
GFU	gas-fired unit
HHV	higher heating value
HR	Heat rate
NR	Newton-Raphson method
OPF	optimal power flow
P2G	power-to-gas

Subscripts

a	Average
b	Base
c	Compressor
g	Generated
i	Node index
j	Node index
k	Link index
l	Load
gfu	Gas-fired unit
p2g	Power-to-gas

Greek symbols

α	Coefficient
β	Coefficient
Δ	Difference
δ	Voltage angle
η	Efficiency
γ	Coefficient
ι	Imaginary unit
ω	Angular frequency

Matrices and vectors

$\boldsymbol{\tau}$	Vector of gas load of gas-fired compressor stations
$\boldsymbol{e}$	Vector of gas injected by power-to-gas units
$\boldsymbol{q}$	Vector of mass flow rates through branches
$\boldsymbol{w}$	Vector of gas injections at each node
A	Connectivity matrix of the gas network's branches
E	Connectivity matrix of the power-to-gas units
T	Connectivity matrix of the gas-fired units
U	Connectivity matrix of the gas compressors

Roman symbols

b	Susceptance, the imaginary part of y
C	Pipe constant
c	Parameter for a gas-fired unit
f	Friction factor of a pipe
G	Volume of gas
g	Conductance, the real part of y
I	Current
L	Length of a pipe or line
P	Active power, the real part of S
p	Pressure
Q	Reactive power, the imaginary part of S
q	Gas flow rate
r	Compressor ratio
r	Resistance, the real part of z
S	Complex power, given by $P + \iota Q$
T	Temperature
t	Time
x	Reactance, the imaginary part of z
y	Admittance, given by $g + \iota b$
z	Impedance, given by $r + \iota x$

Superscripts

k	Newton-Raphson iteration

Other symbols will be introduced as needed.

1 Introduction

1.1 Background

An increasingly large part of European energy is provided by renewable sources such as wind and solar. While they are more sustainable than fossil fuels, the energy they provide fluctuates depending on the weather conditions [1]. On the other hand, in electricity networks, supply and demand must be balanced for stability [2]. This means that ways must be found to absorb the excess renewable energy when supply exceeds demand, and to inject extra energy into the network when supply is insufficient. Gas-fired power plants can provide the latter [3]; more recently, power-to-gas technology has been studied as a way to store surplus renewable electricity in the form of hydrogen or synthetic methane [4].

Both of these technologies imply that power and gas networks are increasingly interdependent. Thus, it is useful to model them together, be it for day-to-day dispatch of energy sources or for long-term infrastructure planning.

1.2 Project goal

In the Infrastructure Outlook 2050 [5], Gasunie and TenneT studied different scenarios for the evolution of energy supply and demand. The idea was to obtain hourly snapshots of the power and hydrogen networks over an entire year. This represents 8760 individual calculations, so a computationally inexpensive model was needed. A linear model based on the transport moment was used for both electricity and hydrogen. While it was sufficient to get a general idea of the gas network behaviour, it proved inaccurate for electricity. Therefore, the object of this thesis is to construct an improved model for the combined network, without losing speed of computation. To this end, in this literature study, we take a look at the models used for individual gas and power networks, as well as various coupling strategies and optimization techniques. We will take a particular interest in linear models, since this is what we are looking to implement.

1.3 State of research

In the early 2000s, a few foundational papers were written, such as [3, 6]. All of them studied natural gas networks and electricity networks with gas-fired power plants. Also commonly cited is [7], which introduced the concept of energy hub as a way to couple general multi-carrier energy systems. Since then, many other papers have been written, studying various aspects of the problem. These aspects range from pure network modelling to infrastructure and market planning.

The majority of the papers cited in this review study natural gas networks and gas-fired power plants. This is not the focus of this work, but as the physics of natural gas and hydrogen are much the same, these sources are still interesting to consider. Some more recent papers such as [8, 9], on the other hand, specifically model power-to-gas facilities under the constraints of renewable energies. As the topic of combined energy networks gains importance, literature reviews [1, 10] as well as textbooks [2, 4] have also been written.

1.4 Scope

The goal of this project is to construct a functional (if simplified) coupled network model. Hence, we will focus exclusively on sources dealing with coupled gas and electricity network models. They often involve optimization, since the systems studied are underdetermined. Many papers also deal with heat networks, renewable energies, water networks, etc.; all of these are outside the scope of this literature study.

The structure of this report is as follows: In section 2, we look at the models used for gas and electricity separately, then in section 3, at the ways the two networks are coupled. Section 4 deals with the numerics of the complex systems of equations involved. Section 5 is a brief tour of the test cases used in the literature or selected for the needs of the project, as well as some useful software. Finally, section 6 discusses the findings.

2 Decoupled gas and power flow

All parts of the combined network need to be modeled. Some, such as transport lines, are described with a governing equation, while others are given in the form of a constraint on the network. This is especially true in the papers where the goal is optimization of the network.

2.1 Power network models

Most of the models studied here are based on AC power flow. Thus, we start by writing down all the elements needed to describe AC networks. This will later clarify how the linear approximation (DC power flow) is constructed. The following is a standard explanation of AC circuits, which is given in many references such as [11].

2.1.1 Introduction

We start with current and voltage. In an AC circuit, they are sinusoidal functions written as follows:

$$\begin{aligned} i(t) &= I_{\max} \sin(\omega t + \phi_I) && = \mathrm{Re}\left(I_{\max} e^{\iota \phi_I} e^{\iota \omega t}\right) \\ v(t) &= V_{\max} \sin(\omega t + \phi_V) && = \mathrm{Re}\left(V_{\max} e^{\iota \phi_V} e^{\iota \omega t}\right) \end{aligned}$$

where $I_{\max}$ (in ampere) is the amplitude of current, $V_{\max}$ (in volt) is the amplitude of voltage, ω (in rad/s) is the angular frequency, and ϕ (in rad) is the phase shift.

We will be looking at steady-state models, so the time-dependent term $e^{\iota \omega t}$ is not used. Instead, we employ phasor representation: we use the RMS values of $v(t)$ and $i(t)$,

$$|V| = \frac{V_{\max}}{\sqrt{2}}, \quad |I| = \frac{I_{\max}}{\sqrt{2}}$$

and use the effective voltage and current phasors [12],

$$V = |V| e^{\iota \phi_V}, \quad I = |I| e^{\iota \phi_I}.$$

Using phasors, we can write down instantaneous power as follows [13]:

$$\begin{aligned} p(t) &= v(t) i(t) \\ &= \sqrt{2}|V| \cos(\omega t) \sqrt{2} |I| \cos(\omega t - \phi) \\ &= |V||I| \cos\phi [1 + \cos(2\omega t)] + |V||I| \sin\phi [\sin(2\omega t)] \\ &= P[1 + \cos(2\omega t)] + Q[\sin(2\omega t)]. \end{aligned}$$

P is called *active* or *real* power, and Q is called *reactive* or *imaginary* power. Together, they form the *complex power*,

$$S = P + \iota Q = V I^*, \tag{1}$$

where $(\cdot)^*$ denotes the complex conjugate.

We can now define another set of important quantities for AC circuits: impedance and admittance, as well as their respective components. *Impedance* is the generalization of resistance to AC circuits. It is denoted by

$$z = r + \iota x,$$

where r is the *resistance* and x is the *reactance.* The inverse of the impedance is called *admittance* and is denoted by

$$y = g + \iota b.$$

We have

$$y = \frac{1}{z} = \frac{r}{|z|^2} - \iota \frac{x}{|z|^2}.$$

The components $g = r/|z|^2$ and $b = -x/|z|^2$ are called *conductance* and *susceptance*, respectively. Finally, we write down the AC extension of Ohm's law: we define Y to be the branch admittance matrix of the network, and we have

$$\boldsymbol{I} = Y\boldsymbol{V}, \tag{2}$$

where $\boldsymbol{I}$ and $\boldsymbol{V}$ are the vectors of injected currents at each bus and of bus voltages, respectively.

2.1.2 Electricity nodes

Kirchhoff's current law (KCL) states that the sum of currents entering a node (or bus) must equal the sum of currents leaving it. Combining this with the complex power equation 1 yields the *complex power balance equation*, which is used in most of the papers studied. Conservation of electric power is modelled in two main flavours: for each bus, and for the whole network. For each bus, we take into account all of the sources, loads, and power flowing through the lines connected to that bus. This may be written in the form of complex power, as in [7], or with separate equations for active and reactive power [3, 14, 15, 16]. In complex power form, KCL for an individual bus j reads

$$S_j - \sum_{i \neq j} S_{ij} = 0,$$

where S_j is the power injected at bus j and S_{ij} is the power flowing between buses i and j. In [17], power conservation is written for each bus as well; however, the complex power equation and Kirchhoff's current law are kept separate. In all of these papers, a set of linear equality constraints is thus obtained: one per bus. In contrast, the power balance may also be written as a single equation for the whole network (e.g. [8, 18]):

$$\sum_i P_i^{\text{source}} = \sum_i P_i^{\text{load}}$$

i.e. the total amount of power injected into the network must equal the total load over all nodes i. We also find inequality constraints for each bus. Most models include upper and lower bounds on the power generated (e.g. [19, 9]) or demanded [3], and the voltage at each bus [3, 16]. Once again, constraints on power can be written either in complex form or with two separate equations.

2.1.3 Electricity lines

Power flow in electricity lines may be approximated with AC or DC equations. The former is more accurate, while the latter is linear and thus easier to compute [20]. AC power flow is often written as a pair of equations, for active and reactive flow [14, 17], but in [7] it is given in complex form. It is derived by combining the complex power equation 1 with Ohm's law 2. At bus i, we have

$$S_i = V_i I_i^* \quad \text{and} \quad I_{ij} = Y_{ij} V_j,$$

where I_{ij} is the current in the branch from bus i to bus j. Combining, we get

$$S_i = V_i (YV)_i^* = V_i \left(\sum_{j=1}^{N} Y_{ij} V_j \right)^*$$

Expanding and using phasor notation:

$$\begin{aligned} S_i &= \sum_{j=1}^{N} |V_i||V_j| \, e^{\iota \delta_{ij}} \, (g_{ij} - j b_{ij}) \\ &= \sum_{j=1}^{N} |V_i||V_j| \, (\cos \delta_{ij} + \iota \sin \delta_{ij}) \, (g_{ij} - \iota b_{ij}) \end{aligned}$$

where $\delta_{ij} = \delta_i - \delta_j$ is the difference between the voltage phase angles of buses i and j. Finally, we separate the real and imaginary terms of S_i to get two power flow equations, one for active and one for reactive power:

$$\begin{aligned} P_i &= \sum_{j=1}^{N} |V_i||V_j| \, (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}) \\ Q_i &= \sum_{k=1}^{N} |V_i||V_j| \, (G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}) \end{aligned} \tag{3}$$

We can now state some simplifying assumptions that allow us to construct the DC approximation of power flow (DC PF):

- Shunt (end-of-line) conductance is negligible.
- Series resistance is negligible: on a line from node i to node j, $r_{ij} \approx 0$, so the impedance is $z_{ij} = r_{ij} + \iota x_{ij} \approx \iota x_{ij}$.

This simplifies the active power transfer along a line [20] to

$$P_{ij} = \frac{1}{x_{ij}} V_i V_j \sin(\delta_i - \delta_j).$$

A final two assumptions are added:

- The voltages are constant and both equal to one unit ($V_i = V_j = 1$ p.u.)
- The difference between the voltage angles is small and thus the small-angle approximation $\sin\theta \approx \theta$ can be used.

The DC PF equation is therefore

$$P_k = P_{ij} = \frac{1}{x_{ij}}(\delta_i - \delta_j) = b_{ij}(\delta_i - \delta_j). \tag{4}$$

From this, a matrix relation for the power injections at each node can be derived [20]:

$$\boldsymbol{P} = B\boldsymbol{\delta}, \tag{5}$$

where B is the *bus susceptance matrix*. Now, this can be reformulated so that the change in power flow in each line is easily found based on the change in power injection at each node. The matrix describing this is called *power transfer distribution factor* (PTDF) [20]. For the change in power on line ij due to an injection of power at node m, it is defined as:

$$\Delta P_{ij} = \mathrm{PTDF}_{ij,m}\Delta P_m.$$

It follows that the total flow over that line is:

$$P_{ij} = \sum_m \mathrm{PTDF}_{ij,m} P_m$$

The PTDF matrix is given by

$$\mathrm{PTDF} = B_{\text{line}} B_{\text{bus}}^{-1} \tag{6}$$

where B_{bus}^{-1} is the pseudo-inverse of the bus susceptance matrix (see appendix A). We see, then, that the PTDF matrix may be calculated in full, but this involves the inversion of a full matrix and is thus computationally expensive [20]. However, note that we can rewrite eq. (6) as

$$B_{\text{line}} = \mathrm{PTDF} \cdot B_{\text{bus}} \tag{7}$$

We can then pick a row of interest in the PTDF matrix (corresponding to a branch in the network) and solve the corresponding system of linear equations. In appendix A, we show how this is done and include examples.

Finally, in the Infrastructure Outlook, a linear transport load model was used, similar to the one described later in section 2.2.1. However, this produced results that did not reflect real-life power flow [5]. Thus, the flow equations based on the PTDF matrix are likely a better linear approximation.

Optimization constraints In papers where optimization is used, upper and lower bounds are put on the power flow. This is written for each line for AC power flow, but it may be written in matrix form when the DC approximation is used, as in [9, 18].

2.1.4 Other elements of the electricity network

Generators Generators are the electricity sources which appear in the power balance equations. Given the topic at hand, the generators considered in our sources are most often gas-fired (e.g. [14, 16]), but some papers such as [19] take non-gas generation into account as well. Renewable electricity sources such as wind [8] also provide some power to the network. Generators have capacity limits, so lower and upper bounds are given whenever the generators are explicitly modelled. In addition, for papers which take time integration into account rather than solving for a steady state (e.g. [8, 18]), ramp-up and -down rates are added as constraints.

Slack generators Transmission losses occur in power flow, but they are not known in advance. In order to maintain balance in the electric network, one generator is designated to have a variable output: it is called the *slack generator* or slack bus [11]. This is explicitly modelled in [14] and [15]; the latter uses not one, but a set of slack buses with respective dispatch factors adding up to 1.

2.2 Gas network models

The gas network has several components to be modeled: gas pipelines, gas nodes, compressor stations, and storage. With the exception of gas nodes, which are only governed by mass conservation, all of the components are described in a variety of ways throughout the literature. In practice, gas networks contain other elements, such as valves and reducers [21], but they are not explicitly modeled in the papers studied.

2.2.1 Gas pipelines

A number of pipeline flow equations are given in [22], but only two are used in all of the papers studied. They are called the Fundamental (or General) Flow Equation and the Weymouth equation and are derived from energy balance. In a pipeline which begins at node i and ends at node j, they both give the gas flow as

$$q_{ij} = C_{ij}\sqrt{p_i^2 - p_j^2}. \tag{8}$$

The steady-state gas flow in pipeline ij is thus a quadratic function of the pressures at its end nodes, p_i and p_j. The direction of the flow is always from high to low pressure. The coefficient C_{ij}, or *pipe conductivity*, depends on the choice of equation. For the Fundamental Flow Equation, assuming a horizontal pipeline and isothermal flow,

$$C_{ij} = 3.7435 \times 10^{-3} E \left(\frac{T_b}{p_b}\right) \left(\frac{1}{GT_a LZ}\right)^{0.5} D^{8/3}$$

with E the pipeline efficiency, G the gas gravity, L the length of the pipe, Z the gas compressibility factor, D the pipe diameter. Additional parameters are sometimes used: for instance, [14, 15] take into account the change of elevation in the pipeline.

The Fundamental Flow Equation is used for example in [3, 7, 23] and is one of the most commonly-employed gas flow equations. The Weymouth equation is used to describe high-pressure, high-flow rate, large-diameter networks [22]. We see it employed for example in [8, 19]. It uses a slightly different pipe conductivity,

$$C_{ij} = 1.1494 \times 10^{-3} \left(\frac{T_b}{P_b}\right) \left(\frac{1}{GT_f LZf}\right)^{0.5} D^{2.5}$$

where f is the dimensionless friction factor, and all the other variables are as defined previously. It should be noted, however, that none of the papers cited justify their choice of Weymouth of Fundamental Flow equation. This may be because they are nearly equivalent and used interchangeably in the context of large-scale gas pipelines.

Linear models and linearizations The nonlinear gas flow equations described above make the gas flow problem a challenging one, particularly when optimization is involved. In order to get around this, some researchers opt to use a linear or piecewise linear model instead. For instance, in [19], several piecewise linear approximations of the gas flow functions are constructed and compared. In [8], the gas network is assumed lossless and a matrix analogous to the PTDF matrix is used (see appendix B). This matrix is named *gas shift factor* (GSF) and it gives a linear calculation for pipeline flows, based on mass conservation at gas nodes. Then, the pressure at nodes is computed using the standard gas flow equation, eq. (8). Another option is to use the first-order Taylor expansion of q_{ij} with respect to p_i and p_j, as is done in [24]. We also cite an interesting linear approximation first published in [25] and used in [14]: the gas flow is assumed to be laminar and is given as

$$q_{ij} = L_{ij}(p_i - p_j), \tag{9}$$

where L_{ij} is a linear pressure analog based on the pressure ratio p_i/p_j. Since this pressure ratio is not known ahead of time, an initial value is chosen for L_{ij}. Nodal pressures are then solved simultaneously, since they form a linear system of equations, and the values of L_{ij} are corrected based on the resulting pressures. This is repeated until convergence is achieved. The pressures thus obtained may also be used as an initial value for more elaborate numerical methods. However, the assumption of laminar gas flow could limit the physical accuracy of this method, since gas flow in large pipes is turbulent.

We end this section with the model used in the Infrastructure Outlook [5]. This model is linear by construction: the concept of *transport load* is used, i.e. the product of the amount Q being transported and the distance L over which it is transported. It is then written as

$$T = Q \cdot L.$$

As the length of each pipeline is fixed, the variables are the quantity of gas and the way it is dispatched in pipelines. The quantity of gas in each pipe is constrained by the pipeline's maximum capacity.

Overall, there are many options for linearization. Piecewise linear models are popular when optimization is needed, but they require a lot of care in their construction and implementation. The Taylor expansion model and linear pressure analog are perhaps more interesting, since they are simpler and seem to yield good accuracy. However, it is arguably even better to describe the gas and power network with conceptually close models. This is what was attempted, with mixed results, with the transport load model. On the other hand, working with the PTDF and GSF matrices is more grounded in reality.

2.2.2 Gas nodes

At gas nodes, conservation of mass dictates that the total flow of gas entering must equal the total flow of gas leaving. This includes the gas entering and leaving through the lines connected to that node, as well as sources and sinks (gas injections and gas loads). This may be written in matrix form for the whole network, as follows [3]:

$$(A+U)\boldsymbol{q} + \boldsymbol{w} - T\boldsymbol{\tau} + E\boldsymbol{e} = 0 \tag{10}$$

where

$$A_{ik} = \begin{cases} +1, \text{ if branch } k \text{ enters node } i, \\ -1, \text{ if branch } k \text{ leaves node } i, \\ 0, \text{ if branch } k \text{ is not connected to node } i. \end{cases}$$

$$U_{ik} = \begin{cases} +1, \text{ if the } k \text{ th unit has its outlet at node } i \\ -1, \text{ if the } k \text{ th unit has its inlet at node } i, \\ 0, \text{ otherwise.} \end{cases}$$

$$T_{ik} = \begin{cases} +1, \text{ if the } k \text{ th turbine gets gas from node } i \\ 0, \text{ otherwise.} \end{cases}$$

$$E_{ik} = \begin{cases} +1, \text{ if the } k \text{ th PtG facility sends gas to node } i \\ 0, \text{ otherwise.} \end{cases}$$

A negative gas injection means that gas is taken out of the network (gas load). Strictly speaking, there may be simpler ways to write this, but having all the variables and parameters explicitly written down adds to the clarity of this model. In addition, some models implement upper and lower bounds on the pressure at nodes [19], or on gas supply and load [8, 9].

2.2.3 Compressor stations

Pressure loss occurs in pipelines due to friction. In order to counteract this, compressor stations are installed along gas pipelines. The energy they need may be provided by electricity, or by gas from the network [10]. In the first case, the compressor is a coupling link between the power and gas networks; in the second, it is only a component of the gas system. We only look at the second case here, and will touch briefly on electric-powered compressors in the next section.

Compressors are active elements of the gas network: that is, they can be controlled by the user and are thus used to regulate the gas flow. This can be represented in several ways: we can impose a fixed outlet pressure [8], a fixed pressure ratio [17], or a pressure difference (absolute or relative). Note also that the outlet pressure p_j must be greater than or equal to the inlet pressure p_i; this can be written as an optimization constraint.

The operation of compressors takes energy; in fact, this accounts for a large part (25 to 50%) of the operating costs of the network [26]. A few different functions are used to model this. The most elaborate equation found in the literature is given in [22] and adapted in [3]. Variants of this equation are also used in [15, 14]. It gives the work done by a compressor (in Joule per kg of gas compressed) as follows:

$$W = \frac{286.76}{G} T_i \left(\frac{\gamma}{\gamma - 1}\right) \left[\left(\frac{p_j}{p_i}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

with G the gas gravity and γ the ratio of specific heats of the gas. On the other hand, in [21], we find the compressor power approximated by

$$W = q_{ij} \frac{p_j - p_i}{\frac{2}{3}p_i + \frac{1}{3}p_j}.$$

Once the compressor power is determined, the volume of gas consumed is given by

$$G = W/\text{HHV}.$$

Simpler models are also used: for instance, [7] opts for a linear function of the pressure difference. The gas demand of the compressor is given by

$$G_c = k_c q_{ij}(p_j - p_i)$$

where k_c is the compressor coefficient, which depends on a variety of physical parameters.

Note also that compressors are not always explicitly modelled, or mentioned at all. For instance, [19] makes no mention of compression, while [16] models compressors as "active pipelines" with the outlet pressure bounded below by the inlet pressure. In the Infrastructure Outlook [5], the pressure loss and gains over the pipeline are simply averaged out and treated as a linear pressure loss.

2.2.4 Gas storage

Linepack Linepack is the amount of gas stored in pipelines. It can be used to offset small variations in gas supply and demand [9], since it is immediately available. It can be represented as a function of the average gas pressure $\tilde{p}_{ij}$ of the inlet and outlet nodes of a given pipeline,

$$\text{LP}_{ij} = k_{ij}\tilde{p}_{ij}$$

The coefficient k_{ij}, called *linepack coefficient*, is a characteristic of the pipeline. In [24], a pressure average is calculated to obtain more accurate results than the "usual" mean.

Large-scale storage One major advantage of gas over electricity is that it can be stored in large quantities and over long periods of time [10]. This is part of the appeal of power-to-gas: surplus renewable energy can be stored in gas form for as long as needed. Storage facilities are modeled in a few different ways: they can be treated as a node [16], as a (positive or negative) injection at a node [15], or as a set of constraints on the gas network [9]. In the latter case, the constraints concern the capacity of the storage facility, gas injection rates (in and out), and minimum and maximum pressures in the storage facility.

3 Coupling gas and electricity

Constructing a model for a combined gas and electricity network involves an exchange of information between the two. This most often occurs through a coupling node, a point in the network which involves both forms of energy. In this section, we take a brief tour through the types of coupling nodes used in the literature.

3.1 Gas-fired generators

Gas-fired power plants, or units (GFU) are the type of coupling node found in the earliest literature (see [3, 16]). As they are used in order to offset daily variations in electricity demand, the volume of gas they consumed is modelled as a function of the power output needed, P_g. In a detailed model, the energy demanded by a GFU with index i is given by its heat rate curve,

$$\mathrm{HR} = \alpha_i + \beta_i P_g + \gamma_i P_g^2.$$

The volume of gas consumed then depends on its heating value. In most papers (e.g. [9, 15], the higher heating value (HHV) is used, but [27] uses the lower heating value, which is a smaller number and thus yields a higher gas consumption. Indeed, the volume of gas consumed is given by

$$G_{\mathrm{gfu}} = \mathrm{HR}(P_g)/\mathrm{HHV} = (\alpha_i + \beta_i P_g + \gamma_i P_g^2)/\mathrm{HHV}. \tag{11}$$

However, a linear model is used in [8]: an efficiency coefficient η_{gfu} is used and the power output is given as

$$P_g = \eta_{\mathrm{gfu}} \cdot g_{\mathrm{gfu}}. \tag{12}$$

A piecewise linear version of eq. (11) is also used in [18], which is likely to offer better accuracy but is not as easy to implement.

3.2 Power-to-gas facilities

Power-to-gas technology consists of using electricity to produce hydrogen through electrolysis, or synthetic natural gas through methanation of the previously obtained hydrogen [10]. It allows for surplus renewable electricity to be stored on the short or long term. Hydrogen has the advantage of causing less energy loss upon conversion: going from electricity from hydrogen and burning the resulting hydrogen produces 34-44% of the initial amount of electric energy, while the round-trip efficiency with synthetic methane is 30-38% [4]. However, there is a limit on the amount of hydrogen allowed in natural gas, while SNG is entirely compatible with the existing natural gas infrastructure. This means that hydrogen necessitates its own transport network [4].

Since the technology is only starting to become economically viable [1], all the sources for this part (e.g. [8, 10, 27, 28]) are very recent. All of them use a linear relationship for power-to-gas modelling. The volume of gas obtained is thus represented by

$$G_{\mathrm{p2g}} = \eta_{\mathrm{p2g}} \cdot P_l/\mathrm{HHV} \tag{13}$$

where P_l is the power load of the P2G facility.

This does not change whether the gas in question is hydrogen or methane. The efficiency and heating value will be different; hydrogen has a lower heating value than methane, thus a larger volume is necessary to obtain the same amount of power.

3.3 Compressor stations

Compressor stations may be powered by electricity; in that case, they act as a coupling node between the gas and the electricity networks. The energy they consume is then converted into a load on the electricity network. The models used for this energy consumption are given in section 2.2.3; all that is left to do is give the resulting power consumption.

3.4 Energy hubs

Energy hubs are occasionally used for general multi-carrier energy systems. In fact, they are an interesting way to represent systems with more than 2 energy carriers. They were first introduced by [7] and they serve to represent a set of energy exchanges: an energy hub may contain sources, loads, converter devices, and storage facilities. The system is then represented as a network of energy hubs. Converter devices are represented with a coupling efficiency between each energy carrier. This efficiency may or may not be linear.

3.5 Constructing a combined model

3.5.1 Without optimisation

After combining the elements of the network, it remains to solve a system of equations: power flow, power balance at nodes, gas flow, gas balance at nodes, coupling node equations. This is generally a system of nonlinear equations which cannot be solved analytically, hence the use of numerical methods. This strategy is used in [14, 15, 17]. The system of equations is generally undetermined. An initial guess is provided by the user, which contains enough data that the solution based on that guess is unique. However, it is not necessarily optimal

[26]. In addition, finding a reasonable value for such a guess is challenging, as discussed in the next section.

3.5.2 With optimisation

An underdetermined system of equations is obtained from all the governing equations listed above. Constraints are put on each component of the network (some are listed throughout the previous sections). The objective function is often the fuel cost (to be minimized), but other choices are possible [26, 7], such as throughput maximization, profit maximization, loss minimization, or pollution minimization. This generally nonlinear system is then solved using a variety of methods.

4 Numerics and optimization algorithms

All of the equations discussed above serve to construct a combined power flow problem. In this section, we look at some techniques used to obtain a snapshot of a combined energy network. They can be divided into two families: with and without optimization.

4.1 Solving without optimization

When the system of equations given by the models is fully determined, it can be solved without optimization.

4.1.1 Solving nonlinear systems: the Newton-Raphson method

We have seen that many of the equations used to describe gas and power flow are nonlinear. Thus, numerical methods are needed to solve the resulting large systems of equations, and the Newton-Raphson method (NR) is a common choice. It is used to solve equations of the form $F(x) = 0$. Starting with an initial guess x_0, each new iteration x^{k+1} is calculated from a first-order Taylor approximation of the function. A correction vector Δx is obtained by solving

$$-J(x^k)\Delta x = F(x^k)$$

where $J(x)$ is the Jacobian matrix (note that this requires the Jacobian to be nonsingular at x^k). Then, the new iteration is given by $x^{k+1} = x^k + \Delta x$.

The Newton-Raphson method has some major advantages: it is straightforward to derive and implement, and it displays quadratic convergence provided the initial guess is sufficiently close to the solution [29]. However, a "good enough" initial value can be difficult to obtain, particularly for gas networks [25]. This can lead to convergence problems. In [25], several ways to formulate gas loadflow problems to be solved with NR are described: based on mass conservation at nodes (Newton-nodal), or based on mass conservation within a loop of the network (Newton-loop). While the former is easy to implement, it has poor convergence properties; on the other hand, the latter has faster convergence and less sensitivity to initial conditions, but it is much more difficult to formulate and requires optimization of the choice of loops.

The Newton-Raphson method is used as-is for combined gas and power flow in [15, 17, 27]. In [14], it is used to solve for power flow, while gas flow is linearized; the two parts of the network then get solved iteratively, one after the other. In [16], NR also serves to solve for power flow, and gas flow is optimized using interior-point linear programming. Both of these examples show that the same technique need not be used for the two parts of a combined network: solving them separately using an appropriate method for each is also an option. Finally, we note that [12] studied Newton-Krylov methods for power flow, as a way to improve on NR's performance and properties.

4.1.2 Linear and partly-linear systems

We have seen in previous sections that linear approximations exist for both gas and electricity. These can be solved with direct methods, but by nature they offer less accuracy than nonlinear models. They can be combined with a more accurate, nonlinear model in order to improve the overall accuracy, as in [14]. This paper uses a linearization for gas flow first described in [25]. The method in question involves a modified pipe coefficient, which is corrected iteratively by solving the linear system several times. This is another way to improve accuracy for a linear approximation.

4.2 Strategies for optimization

In many cases, the variables of the combined network problem are not fully determined. This can occur when the network contains loops. In that case, there exist many solutions to the problem and optimization is necessary. The objective function is usually the cost of operation of the network.

4.2.1 Linear optimization

Linear optimization is the most straightforward type, but it can only be used if the objective function and all the constraints are linear. When this is the case, the problem can be written as

$$\begin{array}{ll} \text{Find a vector} & \mathbf{x} \\ \text{that maximizes} & \mathbf{c}^T\mathbf{x} \\ \text{subject to} & A\mathbf{x} \leq \mathbf{b} \\ \text{and} & \mathbf{x} \geq \mathbf{0} \end{array}$$

where $\mathbf{x}$ is the variable, $\mathbf{b}, \mathbf{c}$ are given vectors, and A is a given matrix.

Linear optimization is used in the Infrastructure Outlook, where the objective function to be minimized is the transport load. However, linear models are not a popular choice due to their low accuracy, so linear optimization is seldom used in practice for energy network modelling.

4.2.2 Nonlinear optimization

Since we are dealing with nonlinear, nonconvex models, appropriate optimization algorithms must be chosen. Nonlinear optimization is a tricky problem; convergence and optimality are difficult to obtain. Therefore, it is not surprising that few authors opt for this solution. Of the sources used here, only [3, 16] go through an optimization step with no prior linearization.

4.2.3 Piecewise linear optimization

A common strategy in our sources is to construct a piecewise linear approximation of nonlinear functions, then use mixed integer linear programming (MILP) to optimize this approximation [19, 9, 8, 18]. MILP is a subset of linear programming, where some of the unknowns are integers. In this case, they are binary variables, which serve to "turn on" or "turn off" the appropriate linear segments approximating a function. This can be done for gas flow or energy conversion, while power flow is usually modelled with the DC approximation. MILP can also be used in combination with another method. This is the case in [9], where MILP is used for the gas flow while another algorithm is used for power flow. The two are then iteratively solved. In [19], several piecewise linearizations of the gas flow are optimized with MILP and compared, but this time the power flow is modelled with the DC approximation and is thus linear.Finally, in [21], sequential piecewise linear programming (SPLP) is used. It consists of multiple iterations of MILP.

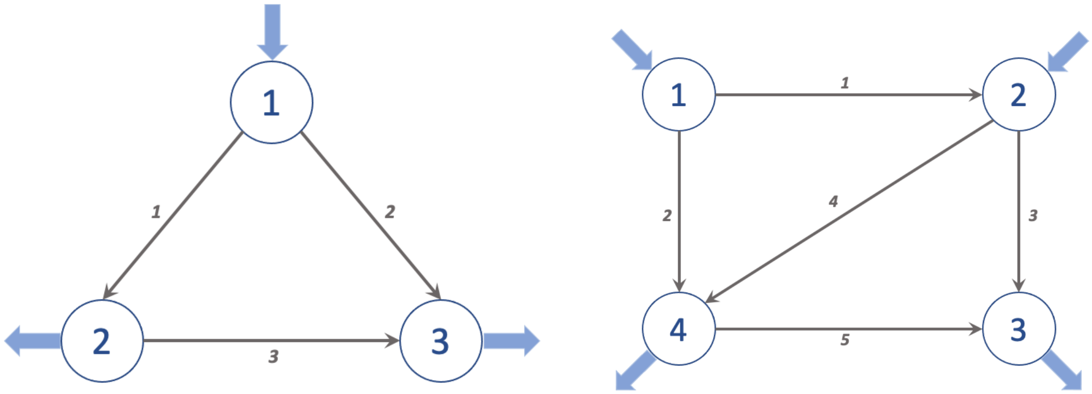
5 Practical aspects: case studies and software

Once constructed, the network models and solving strategies described in earlier sections need to be tested and the results analysed.

5.1 Test networks

Standard IEEE test cases are used as the power network in case studies throughout the literature. In order to construct a combined network, a gas network is linked to it at one or more points. On the other hand, there is no standard for test cases of gas networks, so the papers studied here use many different ones. A variety of small networks are thus used; the only reoccurring one is a model of the Belgian gas network [8, 15, 16], since the data for this one is publicly available. However, a library of gas networks has recently been collected [30] under the name GasLib; it contains some small networks for testing purposes as well as some large ones based on real-world data.

In many papers, two case studies can be found: one with a very small network and a second one with a larger, more realistically-sized network. The smaller power network, in the literature studied, is one with 6 to 14 buses, while the larger one has 30 or 118 buses. As for the gas network, the sizes used range from 6 to 20 nodes. The number of interaction points



(a) 3-node, 3-branch network

(b) 4-node, 5-branch network

Figure 1: Minimal 1-loop and 2-loop networks to be used for testing

varies as well: from just one gas-fired generator to one power-to-gas facility plus up to 9 gas-fired generators. The larger network commonly has extra features as well, e.g. compressors can be added to the gas network [14] or more points of interaction are added between the gas and electric networks [9, 8].

Small test networks like the ones described above are useful for initial model testing. However, one could conceivably run tests on even smaller networks than the ones mentioned above; for example, the smallest possible network containing a loop for each energy carrier is one with 3 nodes and 3 branches. Such tiny networks may be studied analytically to an extent, which is helpful for checking that a model is correctly implemented. In this study, we will use the ones shown in fig. 1 as examples for both gas and power flow.

Eventually, the goal is to apply the model to the Netherlands' large-scale transportation network, as shown in fig. 2.

5.2 Software

The choice of software used to implement each model depends, of course, on the strategy used. For models where optimization is necessary, commercial solvers are often used. Matlab is also a common choice since it is powerful and flexible; in particular, the Matpower library [31] was developed to study power networks.

6 Conclusions and discussion

6.1 Summary

We have explored many ways to formulate and solve steady-state flow problems in combined gas and electricity networks. The goal of the project is to formulate a linear model; however, it is instructive to look at more elaborate ones, as they offer insight into the challenges of

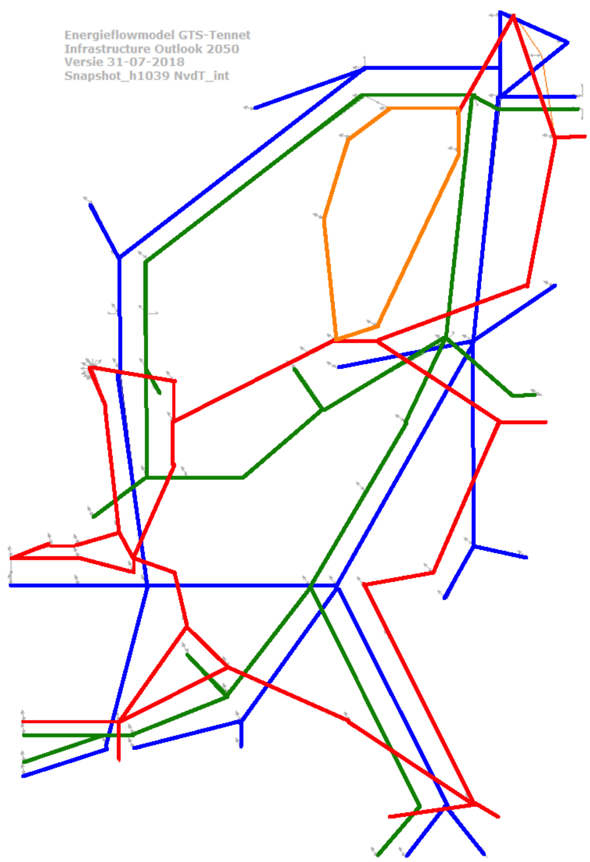


Figure 2: Potential combined network of the Netherlands in 2050. Red/orange = electricity; blue = hydrogen; green = methane. (Diagram provided by Gasunie)

network modelling. While many authors use nonlinear models, which offer better accuracy, some opt for linearization, or for the construction of linear models as a first intention. In this case, care must be taken to state the simplifying assumptions clearly and to construct an accurate linearization. For power flow, there is a single standard linear model, but this is not the case for gas flow. We have thus encountered a variety of linear gas flow models; further investigations will be needed to determine which is the most appropriate for the needs of the project. As for interaction between networks, each component can be described with a linear model or a constant, but so far we have no description of how to calculate the parameters involved. Finally, a range of solving strategies has been described; they depend in part on the model formulation, but also on the desired numerical properties. Perhaps the most important thing to remember about these strategies is that we need not use the same one for both halves of a combined network. The challenge, then, is to maintain good communication between the different parts.

6.2 Discussion

The study of combined gas and power networks is a recent field; the oldest of the references used here was published in 2003. More recent still is the study of networks including power-to-gas, so literature is still relatively sparse. In addition, many authors focus on aspects which are not directly relevant for this project, such as day-to-day scheduling or market planning. For this reason, they need high-accuracy models, and linear ones are not suitable for their purposes. On the other hand, this project will focus on the feasibility of scenarios for future infrastructure, and thus a first-order approximation is sufficient for now. Still, these papers are interesting to study, since many of the governing principles are the same. For instance, comparing the different models lets us know which constraints are absolutely necessary and which can be simplified. We should also note that our sources focus almost exclusively on power-to-gas technology producing synthetic natural gas, rather than hydrogen, due to the infrastructure constraints mentioned in section 3. The physics are the same, but some numerical values change and should be carefully checked when implementing the future model.

6.3 Potential research questions

In the work following the present literature study, we will be constructing a linear model for a combined hydrogen and power network in an attempt to find out which scenarios for infrastructure are feasible. Therefore, we will start by investigating the following questions:

- How can we formulate a unified linear model for the combined network?
  - Which physical constraints must be respected and which can be simplified?
  - How can the model reflect the different reaction times of the gas and power network?

- Can and should the transport load model be used for the gas network?
  - Is the transport load formulation compatible with the use of gas shift factors?
  - If we choose to model the gas network using transport load, how can we model the communication between the gas and power network?
- How much does such a unified model improve on the one used in the Infrastructure Outlook?

Input data will also be needed, which prompts us to ask:

- How much data is needed to run a simulation?
- What is a reasonable set of input data?
- How can we tell whether a set is realistic?
- What are the consequences of a bad choice of input data?

Finally, we will be examining the results of simulations. We can then attempt to answer the following questions:

- How accurate are the results?
  - Can an error estimate be obtained?
  - How can we judge the accuracy and usefulness of the results?
  - How much accuracy is needed for the needs of the project?
- Which scenarios for infrastructure are feasible and desirable?
- Which scenarios for energy generation are feasible and desirable?
- How much interaction should there be between the gas and power networks? Where and when should it occur?
- How and how much can power-to-gas technology alleviate the needs for expansion of electric infrastructure?

Further questions may arise during the research process, and the above ones may not all be answered during the project. However, this is a good place to start.

A Computing the PTDF matrix

The following equations are taken from [20]. Recall that the PTDF matrix is given by

$$B_{\text{line}} = \text{PTDF} \cdot B_{\text{bus}}$$

The bus susceptance matrix B_{bus} is constructed as follows:

- The diagonal elements B_{ii} are the sum of all the line susceptances of the lines connected to node i, $B_{ii} = \sum_j b_{ij}$
- Off-diagonal elements:
  - If there is a line between i and j, $B_{ij} = -b_{ij}$.
  - Otherwise, $B_{ij} = 0$.

Note that the number of rows and the number of columns correspond to the number of buses in the network. This is always a square matrix, but it is singular. Its pseudo-inverse B_{bus}^{-1} is calculated as follows:

- Remove the row and column corresponding to the slack bus.
- Invert the resulting matrix.
- Insert a row and a column of zeros at the row and column corresponding to the slack bus.

The line susceptance matrix B_{line} is the one used in eq. (5): it relates the flow in each line with the voltage angles. It has dimension $L \times N$, where L is the number of lines and N is the number of nodes. (In general, it is not square.) Its elements are given by:

$$B_{kj} = \begin{cases} b_{kj}, \text{ if line } k \text{ leaves node } j \\ -b_{kj}, \text{ if line } k \text{ enters node } j \\ 0, \text{ if line } k \text{ is not connected to node } j \end{cases}$$

Every row then only has two nonzero elements, at the columns corresponding to the nodes that each line connects. The network is treated as a directed graph.

A.1 Three-node network

The illustrations from section 5 are shown again here for convenience. For clarity, in this subsection and the next, we refer to branches by the indices of the nodes they connect (e.g. b_{12} is the susceptance of the branch connecting nodes 1 and 2). We will order them according to the numbering visible in fig. 3.

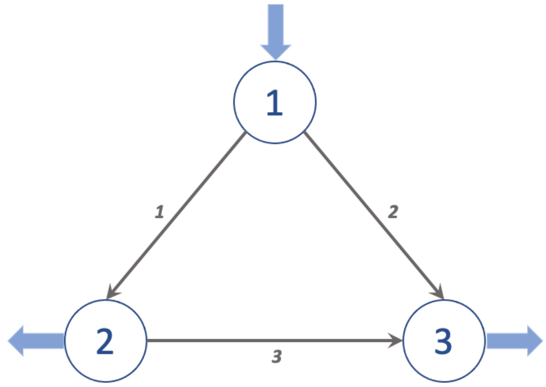


Figure 3: 3-node, 3-branch power network

Then, for the 3-node network, we have branches (12), (13), (23), respectively numbered 1, 2, 3. Following the construction given above, the bus susceptance and line susceptance matrices of the 3-node network are respectively:

$$B_{\text{bus}} = \begin{pmatrix} b_{12}+b_{13} & -b_{12} & -b_{13} \\ -b_{12} & b_{12}+b_{23} & -b_{23} \\ -b_{13} & -b_{23} & b_{13}+b_{23} \end{pmatrix} \qquad B_{\text{line}} = \begin{pmatrix} b_{12} & -b_{12} & 0 \\ b_{13} & 0 & -b_{13} \\ 0 & -b_{23} & b_{23} \end{pmatrix}$$

As an example, let us set $b_{12} = 5, b_{13} = 8, b_{23} = 20$. We will assume that node 3 is the slack bus and compute first the full PTDF matrix using eq. (6), then its first row by solving eq. (7). We have:

$$B_{\text{bus}} = \begin{pmatrix} 13 & -5 & -8 \\ -5 & 25 & -20 \\ -8 & -20 & 28 \end{pmatrix} \qquad B_{\text{line}} = \begin{pmatrix} 5 & -5 & 0 \\ 8 & 0 & -8 \\ 0 & 20 & -20 \end{pmatrix} \tag{14}$$

and

$$B_{\text{bus}}^{-1} = \begin{pmatrix} 1/12 & 1/60 & 0 \\ 1/60 & 13/300 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

hence

$$\text{PTDF} = B_{\text{line}} B_{\text{bus}}^{-1} = \begin{pmatrix} -1/3 & 2/15 & 0 \\ 2/3 & 2/15 & 0 \\ 1/3 & 13/15 & 0 \end{pmatrix}$$

Suppose now that we are only interested in the first row of the PTDF matrix; that is, we want to know the change in power flow in line (12) if power is injected at any given node

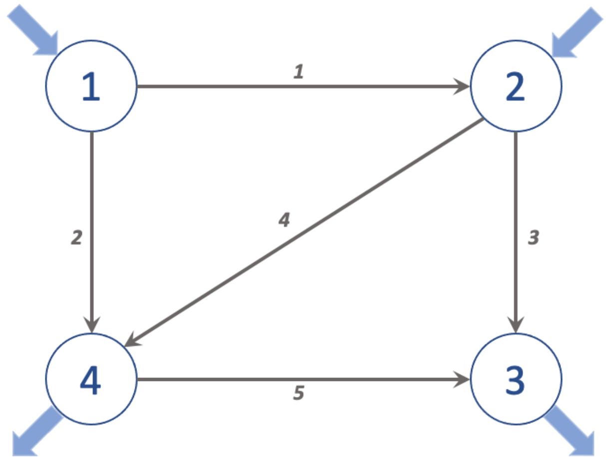


Figure 4: 4-node, 5-branch power network

(and an equivalent withdrawal happens at the slack node). We then need to solve:

$$B^{\text{line}}_{1,\cdot} = \text{PTDF}_{1,\cdot} \cdot B_{\text{bus}}$$

$$\begin{pmatrix} 5 & -5 & 0 \end{pmatrix} = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 13 & -5 & -8 \\ -5 & 25 & -20 \\ -8 & -20 & 28 \end{pmatrix}$$

This has solution

$$\begin{pmatrix} 1/3 + x_3 & -2/15 + x_3 & x_3 \end{pmatrix}$$

and we just need to set $x_3 = 0$ to find the first row of the PTDF matrix we found before. Alternatively, we can remove the row and columns corresponding to the slack bus from all parts of the linear equation:

$$\begin{pmatrix} 5 & -5 \end{pmatrix} = \begin{pmatrix} x_1 & x_2 \end{pmatrix} \begin{pmatrix} 13 & -5 \\ -5 & 25 \end{pmatrix}$$

This has solution $(1/3, -2/15)$ and we just need to insert a 0 at the index corresponding to the slack bus.

A.2 Four-node network

In this network, we have branches (12), (13), (23), (24), (34) numbered 1 through 5 (as shown in fig. 4). The bus susceptance and line susceptance matrices are as follows:

$$B_{\text{bus}} = \begin{pmatrix} b_{12}+b_{14} & -b_{12} & 0 & -b_{14} \\ -b_{12} & b_{12}+b_{23}+b_{24} & -b_{23} & b_{24} \\ -0 & -b_{23} & b_{23}+b_{34} & b_{34} \\ -b_{14} & -b_{24} & -b_{34} & b_{14}+b_{24}+b_{34} \end{pmatrix} B_{\text{line}} = \begin{pmatrix} b_{12} & -b_{12} & 0 & 0 \\ b_{14} & 0 & 0 & -b_{14} \\ 0 & b_{23} & -b_{23} & 0 \\ 0 & b_{24} & 0 & -b_{24} \\ 0 & 0 & 0-b_{34} & b_{34} \end{pmatrix}$$

Note that the bus susceptance matrix contains zeros this time, and the line susceptance matrix is not square.

A.3 Generation shift factors

The PTDF is a generalization of the generation shift factor (GeSF), which is computed from eq. (5) [32]. Indeed, we have the power injections at each node given by

$$\boldsymbol{P} = B\boldsymbol{\delta}$$

and the flows across branches given by

$$\boldsymbol{P}_B = DA\boldsymbol{\delta},$$

where A is the node-arc incidence matrix and D is a diagonal matrix in which $D_{kk} = -b_k$, the negative line susceptance of line k. Combining the two yields

$$\boldsymbol{P}_B = DAB^{-1}\boldsymbol{P}.$$

Hence, for a change $\Delta\boldsymbol{P}$ in power injection at the nodes, we have

$$\Delta\boldsymbol{P}_B = DAB^{-1}\Delta\boldsymbol{P}.$$

The quantity $\Delta\boldsymbol{P}_B$ is the generation shift factor. Once again, since the bus susceptance matrix is not actually invertible, we are using the pseudo-inverse previously described. However, for a large network, these matrices become large as well and inverting B is impractical. Thus, we can also obtain $\Delta\boldsymbol{P}_B$ for a given $\Delta\boldsymbol{P}$, by solving:

$$\Delta\boldsymbol{P} = B\Delta\boldsymbol{\delta}.$$

We can solve for $\Delta\boldsymbol{\delta}$, and thus obtain the change in flow in the lines using

$$\Delta\boldsymbol{P}_B = DA\Delta\boldsymbol{\delta}.$$

B The gas shift factor

The gas shift factor matrix mentioned in section 2 and used in [8] is not explicitly described in that paper. In this section, we attempt to construct a matrix that could fill this role, following a similar process as for the GeSF.

Recall eq. (10), the matrix equation describing mass balance at each node. For a simple network with no compressor, gas turbine or power-to-gas facility, it simplifies to

$$A\boldsymbol{q} + \boldsymbol{w} = 0,$$

where A is the connectivity matrix of the network, $\boldsymbol{q}$ is the gas flow vector and $\boldsymbol{w}$ is the vector of gas injections. Then, if a change in injection Δw occurs at node i, a change in flow $\Delta\boldsymbol{q}$ must occur in the network, such that

$$A(\boldsymbol{q} + \Delta\boldsymbol{q}) + (\boldsymbol{w} + \Delta\boldsymbol{w}) = 0.$$

This implies

$$A\Delta\boldsymbol{q} = -\Delta\boldsymbol{w}.$$

Since the connectivity matrix typically has more columns than rows (corresponding to lines and nodes, respectively), this will in general be an underdetermined system. In addition, it is based on mass balance at nodes, rather than linear flow equations, so this version is not analogous to the PTDF (or to its more specific version the generation shift factor). However, we can push further by using one of the linear gas flow models described in this document: eq. (9). Indeed, if we consider the difference in pressure between the two endpoints of a pipeline as a single variable, we may rewrite it as

$$q_{ij} = L_{ij}(p_j - p_i) = L_{ij}\delta^p_{ij}.$$

In matrix form:

$$\boldsymbol{q} = L\boldsymbol{\delta}^p.$$

In order for this to make sense (particularly in terms of matrix dimensions), L is defined as a diagonal matrix in which L_{kk} is the linear pipe conductivity of pipe k, much like matrix D in the previous section. Combining this with the mass balance equation:

$$\boldsymbol{w} = -A\boldsymbol{q} = -AL\boldsymbol{\delta}^p$$

Just as for the generation shift factor, we can then solve the following equation for $\Delta\boldsymbol{\delta}^p$:

$$\Delta\boldsymbol{w} = -AL\Delta\boldsymbol{\delta}^p \tag{15}$$

From there, we obtain the resulting change in gas flow (the gas shift factor) using

$$\Delta\boldsymbol{q} = L\Delta\boldsymbol{\delta}^p.$$

This is closer to an analog of the generation shift factor. However, using the linear pipe conductivity and pressure difference as direct analogs to the line susceptance and bus voltage angles may be hazardous.

B.1 Example

We take the same 4-node network and set $L = \text{diag}(10, 2, 5, 8, 1)$ and $\Delta\boldsymbol{w} = (1, -1, 0, 0)$. Then:

$$\Delta\boldsymbol{w} = -AL\Delta\boldsymbol{\delta}^p$$

$$\begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} = -\begin{pmatrix} -1 & 0 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{pmatrix}$$

This has solution $\Delta\boldsymbol{\delta}^p = (1/10 - 4/5d_4, -4d_4 - 1/2d_5, 8/5d_4 + 1/5d_5, d_4, d_5)^\top$. However, if we choose $\Delta\boldsymbol{w} = (1, 0, 0, 0)^\top$, the equation has no solution. Surprisingly, if we choose $\Delta\boldsymbol{w} = 0$, we obtain a solution that looks like the first even though there should be no change in the network. All this prompts a few remarks:

- Practically speaking, we expect that the solution will not be unique, since the slow speed of gas in networks creates degrees of freedom. However, this is not reflected in the way we have constructed the gas shift factor: indeed, eq. (15) assumes that the change in pressure differences is immediate following an injection at a node. This is the way power networks work, but it is in fact an unphysical assumption for a gas network.
- Because the equations in this section work the same as DC PF equations, it is not surprising that the choices of $\Delta\boldsymbol{w}$ where the sum of the elements is zero are the ones with a solution: this functions as an "equivalent withdrawal at the slack bus" without which there is no solution. This, combined with the fact that $\Delta\boldsymbol{w} = 0$ yields a nonzero solution, is a further indication that our derivations are not a good reflection of reality.
- Despite the above problems, this is an interesting exercise. The next iteration should start by a careful definition of the constraints at play and the resulting matrix relations.

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