Modelling relative permeabilities and capillary pressures with hysteresis for reservoir simulation

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Problem background

In petroleum reservoir engineering various techniques are used to enhance the oil recovery from a reservoir. In waterflooding water is injected in one or more places (injection wells) in a reservoir under high enough pressure for the oil in the reservoir to be pushed by the injected water towards the producing wells of the reservoir (oil displacement). In water alternating gas (WAG) injection water and gas are injected in turn for the same effect.

Here, we consider a waterflooding in one space dimension. On one end water is injected and on the other end oil and water are produced. We assume oil and water to be incompressible. The two-phase flow model of incompressible fluid flow through a porous medium in one space dimension we consider is given by the transport equations for the phase masses for oil and water (w=water, o=oil),

$$\frac{\partial}{\partial t}\left(\phi \rho_o S_o\right) + \frac{\partial}{\partial x}\left(\phi \rho_o v_o\right) = 0,$$

$$\frac{\partial}{\partial t}\left(\phi \rho_w S_w\right) + \frac{\partial}{\partial x}\left(\phi \rho_w v_w\right) = 0,$$

and Darcy's equations,

$$q_o = -\frac{k k_{ro}}{\mu_o}\frac{dp_o}{dx} = -\lambda_o \frac{dp_o}{dx},$$

$$q_w = -\frac{k k_{rw}}{\mu_w}\frac{dp_w}{dx} = -\lambda_w \frac{dp_w}{dx}.$$

These equations are written in terms of the fluid phase pressures (p_o and p_w) and the fluid phase volume fractions (the saturations S_o and S_w), which need to be solved from these equations given appropriate data. The actual velocities v_i are related to the Darcy (or superficial) velocities q_i by $q_i = \phi v_i$. Additionally, we require that the saturations add to one:

$$S_o + S_w = 1.$$

Due to surface tension the oil and water pressures are not equal. The difference between the two pressures is the capillary pressure, p_c:

$$p_c = p_o - p_w.$$

Furthermore, the reservoir is taken horizontal: the effect of gravity is neglected. The densities ρ_o and ρ_w are constant, as are the porosity ϕ, the absolute permeability k and the viscosities μ_o and μ_w.

The relative permeabilities (or "relperms") k_{ro} and k_{rw} and the capillary pressure p_c are modelled as functions of the water saturation. These functions vary from (oil) field to field and so are not known exactly in advance. In practice, these functions of the saturations are determined in an experimental setup: a piece of rock ("core") is taken from the reservoir at hand and in a laboratory the oil is pushed out from the core by injected water in a controlled experiment. This core flood experiment is done such that a one-dimensional description can be used to model the two-phase flow through the rock. In the one-dimensional model some functional model is assumed for the relative permeabilities and the capillary pressure and the experiment is simulated with that model with estimated parameters. By comparing the outcome of the one-dimensional

model with the experiment it is possible to improve model by adapting the parameters until a best fit is achieved. The resulting parameters are then used in the reservoir simulation model for that reservoir.
A complication is that these curves show a hysteresis effect: relperms and capillary pressures follow different curves as functions of the water saturation depending on whether the water saturation increases or decreases. The curves when going from $S_w = 0$ to $S_w = 1$ and from $S_w = 1$ to $S_w = 0$ are bounding curves in the sense that systems for S_w between 0 and 1 will follow curves ("scanning curves") in between these bounding curves. In the water alternating gas (WAG) process water and gas are injected in turn (instead of only water as in waterflooding) and then this hysteresis effect is particularly important, but also in waterflooding it can have significant effect when an "oil bank" (a zone of higher oil saturation) is moving through the reservoir.

A further complication is that in enhanced oil recovery techniques the relperm and capillary pressure bounding curves are not fixed during a simulation, but actually change in time due to changing fluid compositions.

Assignment

This assignment is about the development of a hysteresis model for two-phase flow that satisfies the physical constraints for relperm and capillary pressure models. Furthermore, it should give a mathematically well-posed two-phase flow model. The numerical model should be robust, stable, and efficient.

In the first part we assume that the bounding curves are fixed. The idea is then to develop a model that allows for relperm and capillary pressure calculations honoring the hysteresis effect with appropriate scanning curve trajectories. One idea is to fix a coordinate system to the bounding curves, which can be used to do the calculations in a consistent way for all coordinate locations between the bounding curves. One purpose of this first part is to show the difference between a model with and without hysteresis.

In the second part we assume the bounding curves to be time dependent. Following up on the idea above the local coordinate system attached to the bounding curves would then be a moving system in time.

The model is analyzed and implemented for the 1D case. The computer program can be prototyped e.g. a Matlab code. Ultimately, it is meant to be implemented in a professional reservoir simulator.

The assignment roughly consists of the following parts:

1. Literature review.
2. Development of a hysteresis model with fixed bounding curves.
3. Implementation and demonstration in a 1D model.
4. Extending the hysteresis model with moving bounding curves.
5. Implementation and demonstration in a 1D model.
6. Implementation in a reservoir simulator.
7. Writing the thesis.

The computer program also needs to be such that hysteresis, relperms and capillary pressure curve are made visible in an easy way.

Literature

[1] K. Aziz; A. Settari: *Petroleum Reservoir Simulation* Applied Science Publishers, London, 1979.