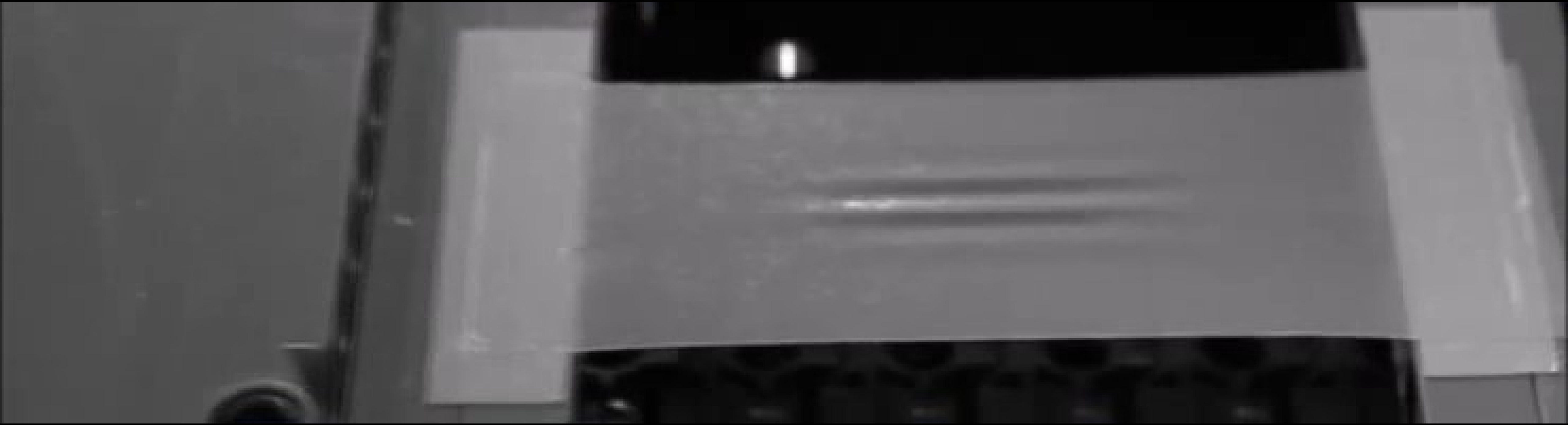
Sipos *et al.* (2016), *Int. J. Solids Struct.*



Modelling Wrinkling Behaviour of Large Floating Thin Offshore Structures

An application of Isogeometric Structural Analysis for Post-Buckling Analyses

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To obtain the degree of Master of Science in Applied Mathematics

TU Delft

Outline

- Research Goal
- Methods
  - Basics of Isogeometric Analysis
  - Kirchhoff-Love shell theory
  - Arc-Length methods
- Results
- Conclusions & Recommendations

Research Goal

Research question:

"How can *wrinkling formation* of large floating thin structures be numerically modelled with *Isogeometric Analysis*?"

Research goal:

"Develop a *geometrically nonlinear* shell model based on *Kirchhoff-Love shell theory* and *Isogeometric Analysis* for *post-buckling analysis*."

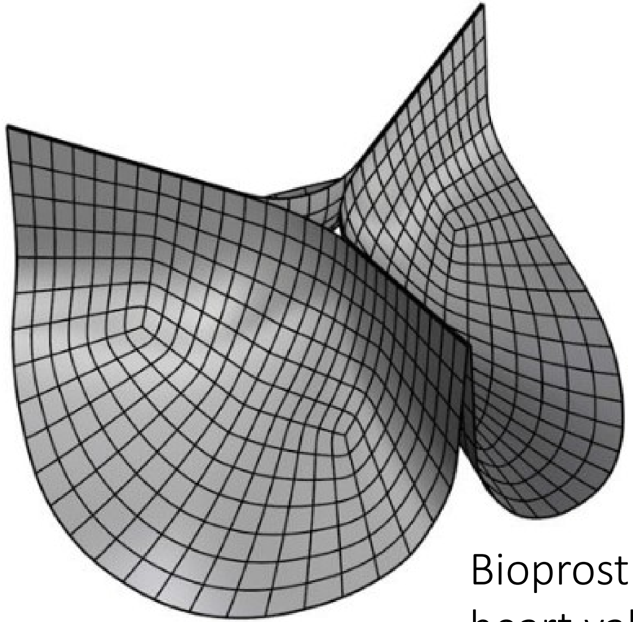
Methods

Basics of Isogeometric Analysis

Basics of Isogeometric Analysis
Philosophy

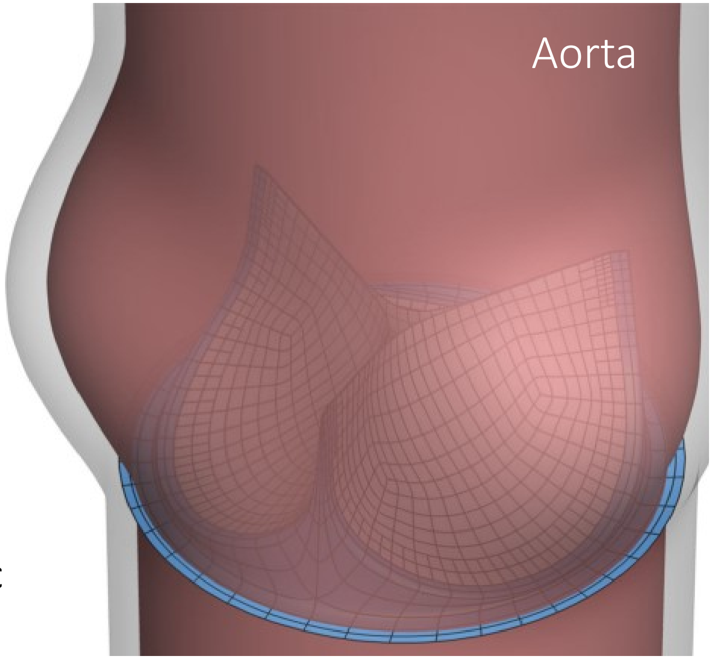
Seamless integration between Computer Aided Design (CAD) and Analysis.

(Parametric) Computer Aided Design

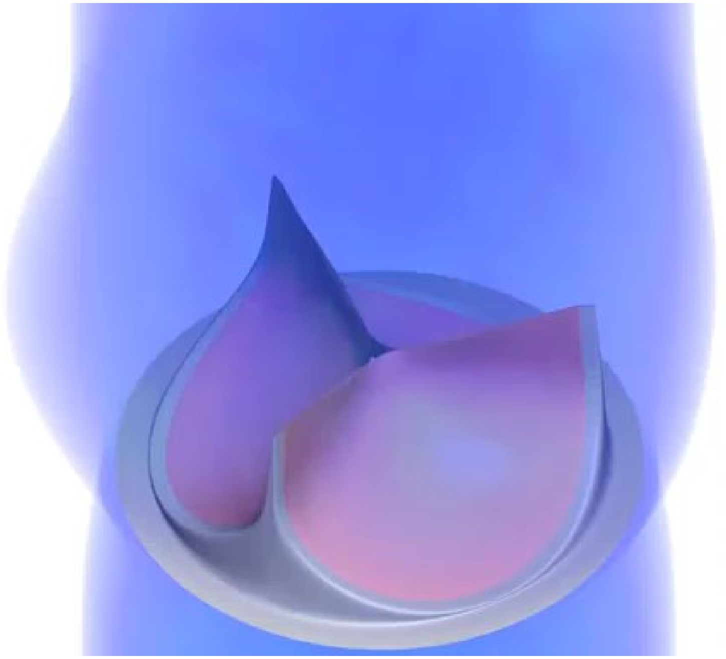


Bioprosthetic
heart valves

Model Domain



Numerical Analysis

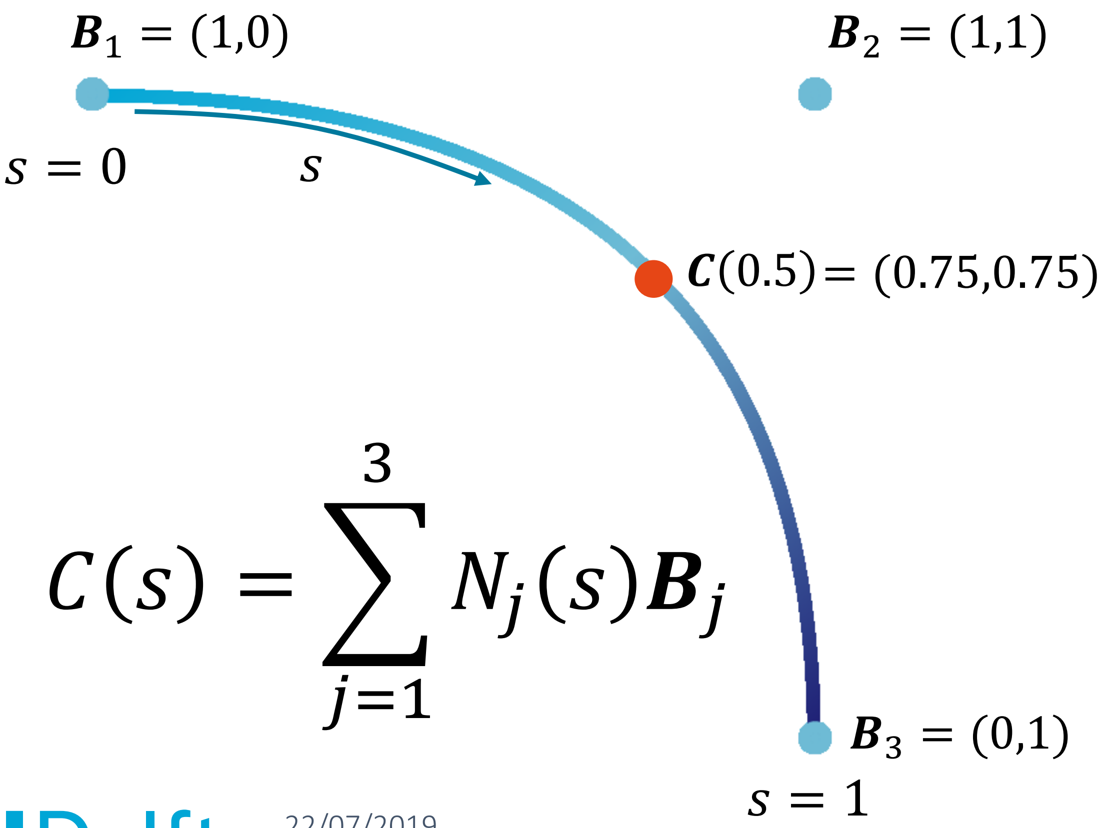


Hsu *et al.* (2015), *Comput. Mech.*

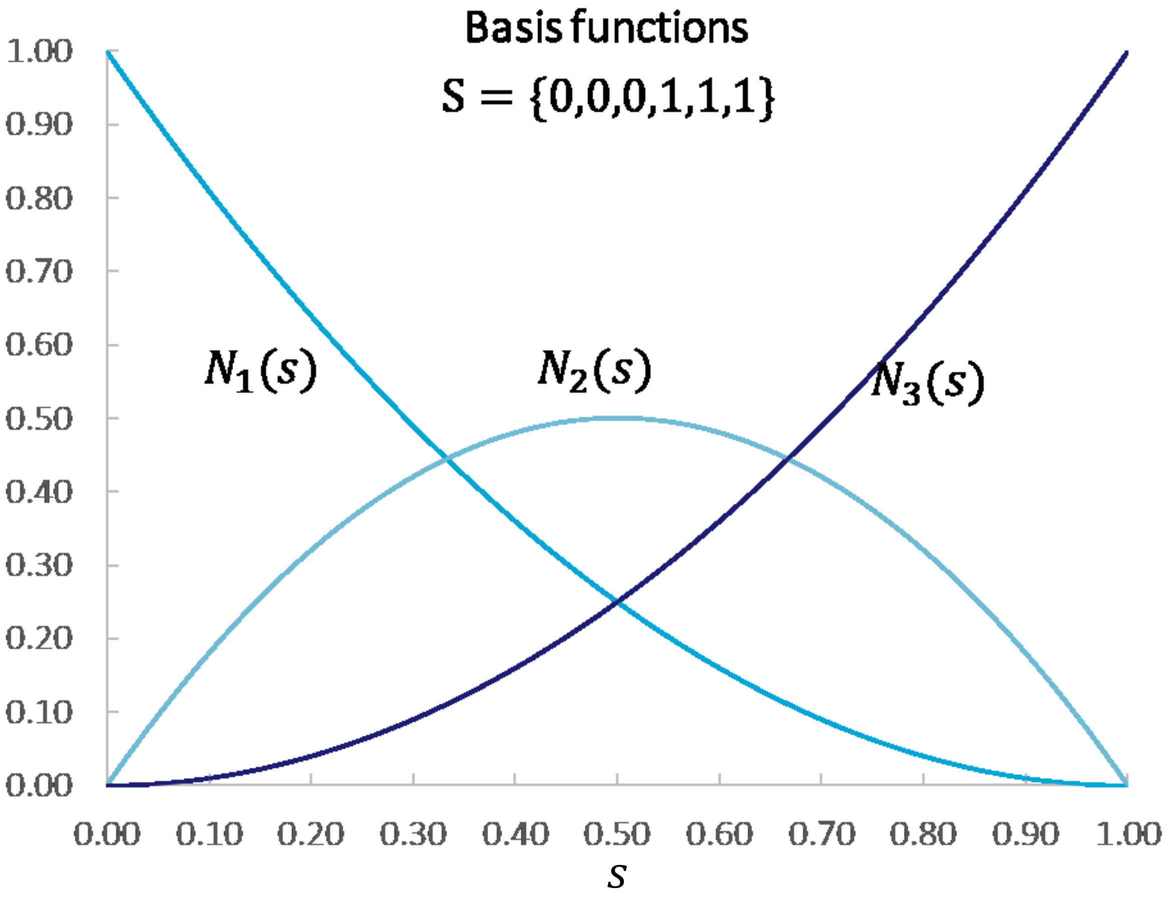
Basics of Isogeometric Analysis

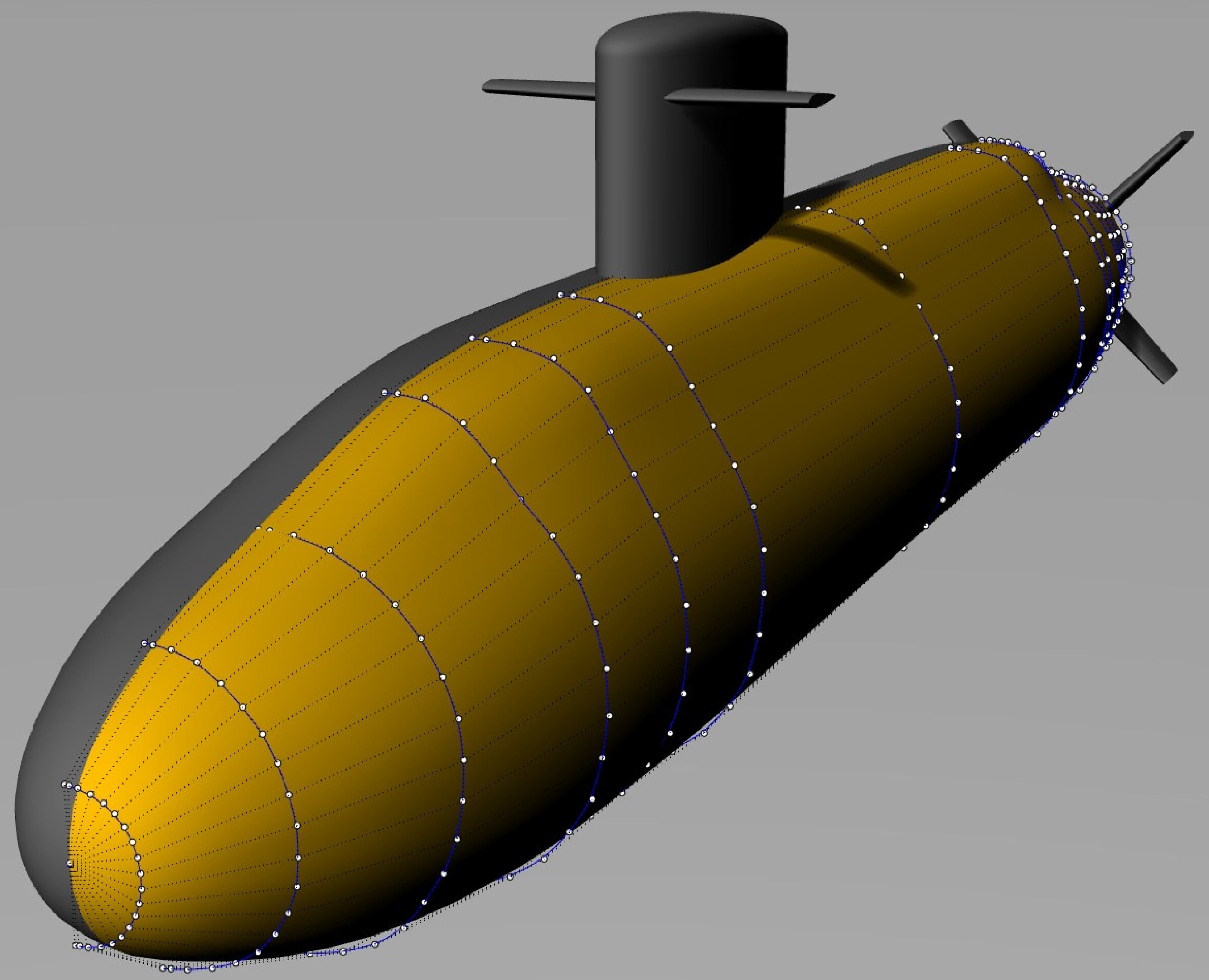
Mathematical Concepts (1)

B-spline curve



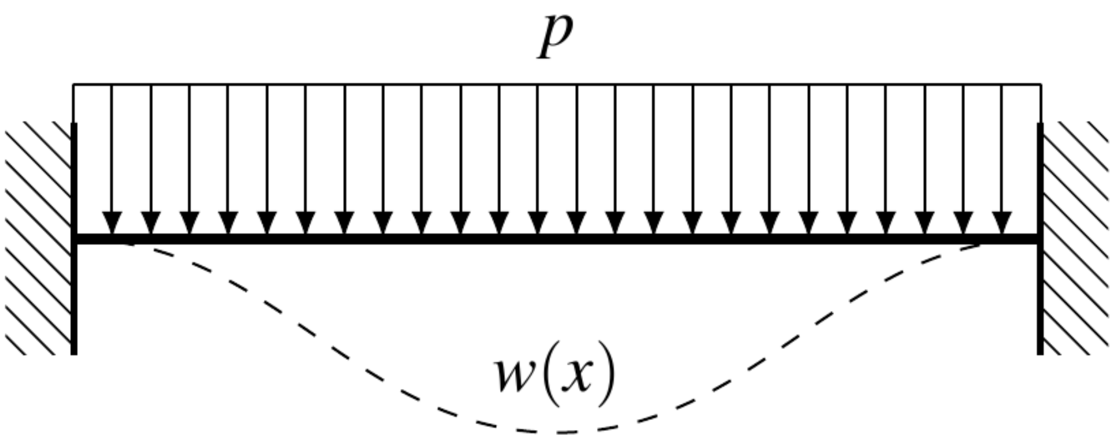
$$C(s) = \sum_{j=1}^{3} N_j(s)\boldsymbol{B}_j$$





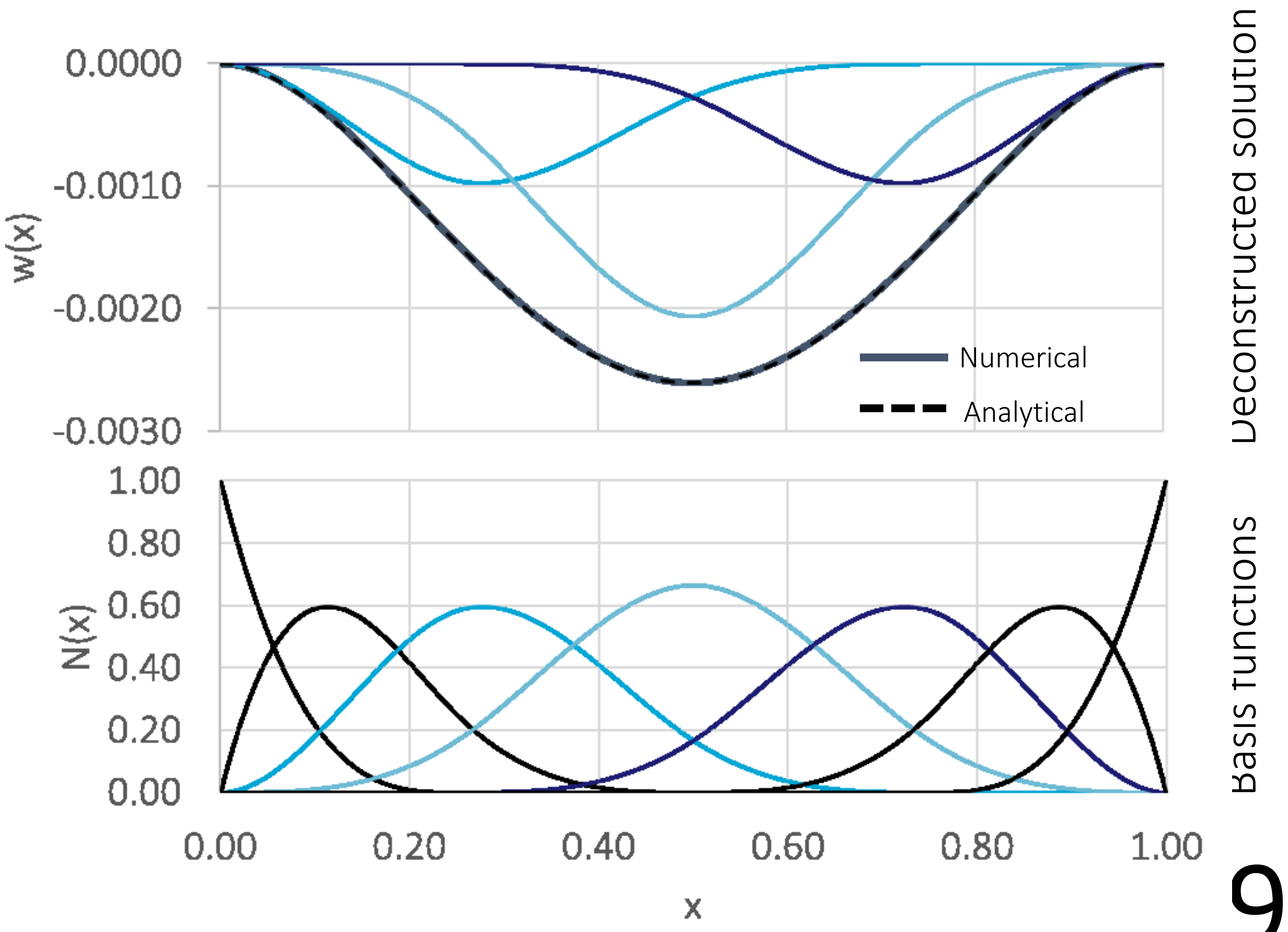
Basics of Isogeometric Analysis

Mathematical Concepts (2)



$$w(x) \approx \sum_{j=1}^{N} \alpha_j \, N_j(x)$$

k-refinement Piecewise-polynomial basis (p = 3)



Methods

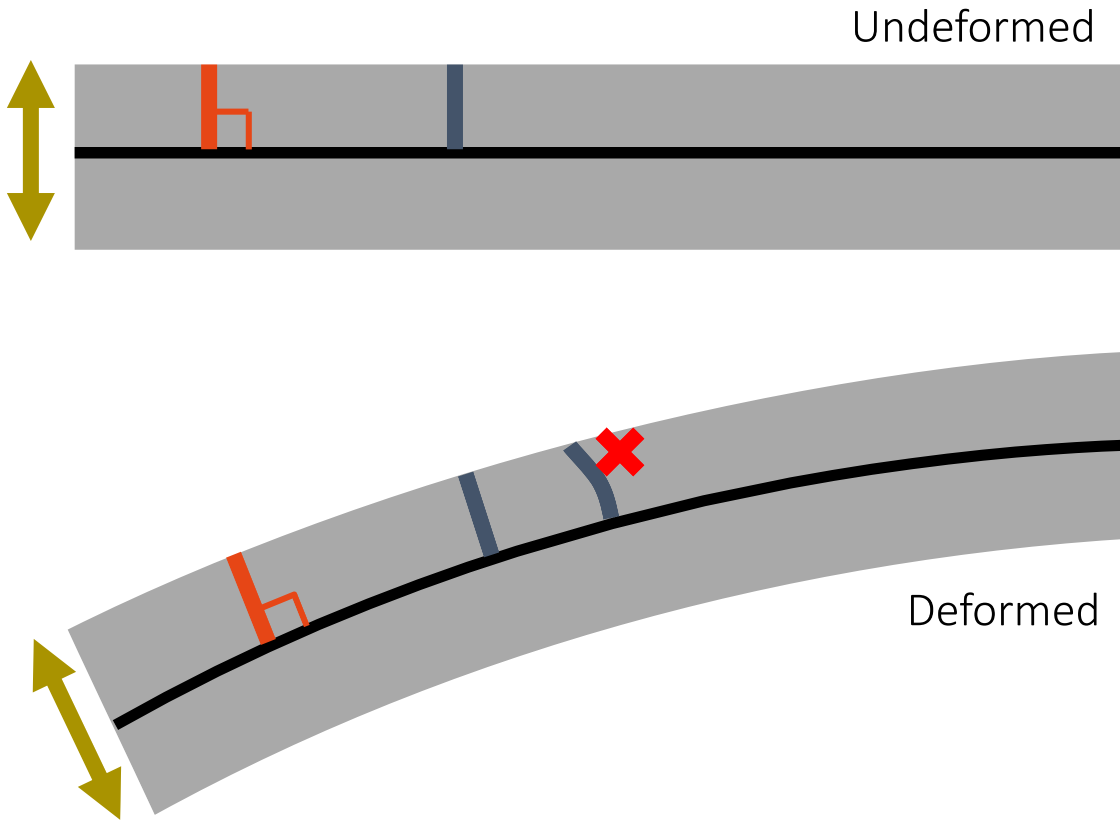
(Geometrically Nonlinear) Kirchhoff-Love Shell Formulation

Isogeometric Shell Model

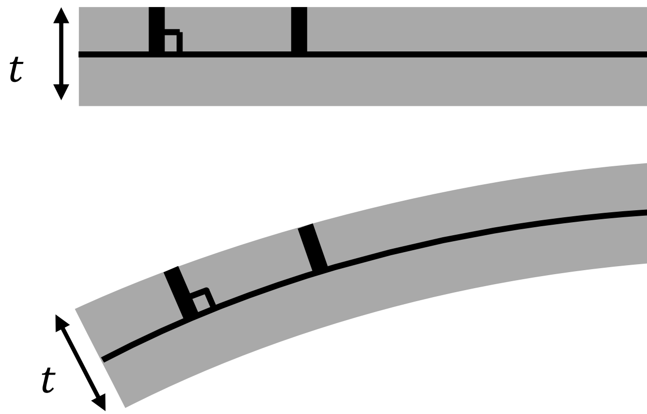
Kirchhoff-Love Shell Model

1. Straight lines normal to the mid-surface remain normal to the mid-surface after deformation
2. Straight lines normal to the mid-surface remain straight after deformation
3. The thickness of the shell does not change during deformation

Valid for low thickness vs. geometry dimensions

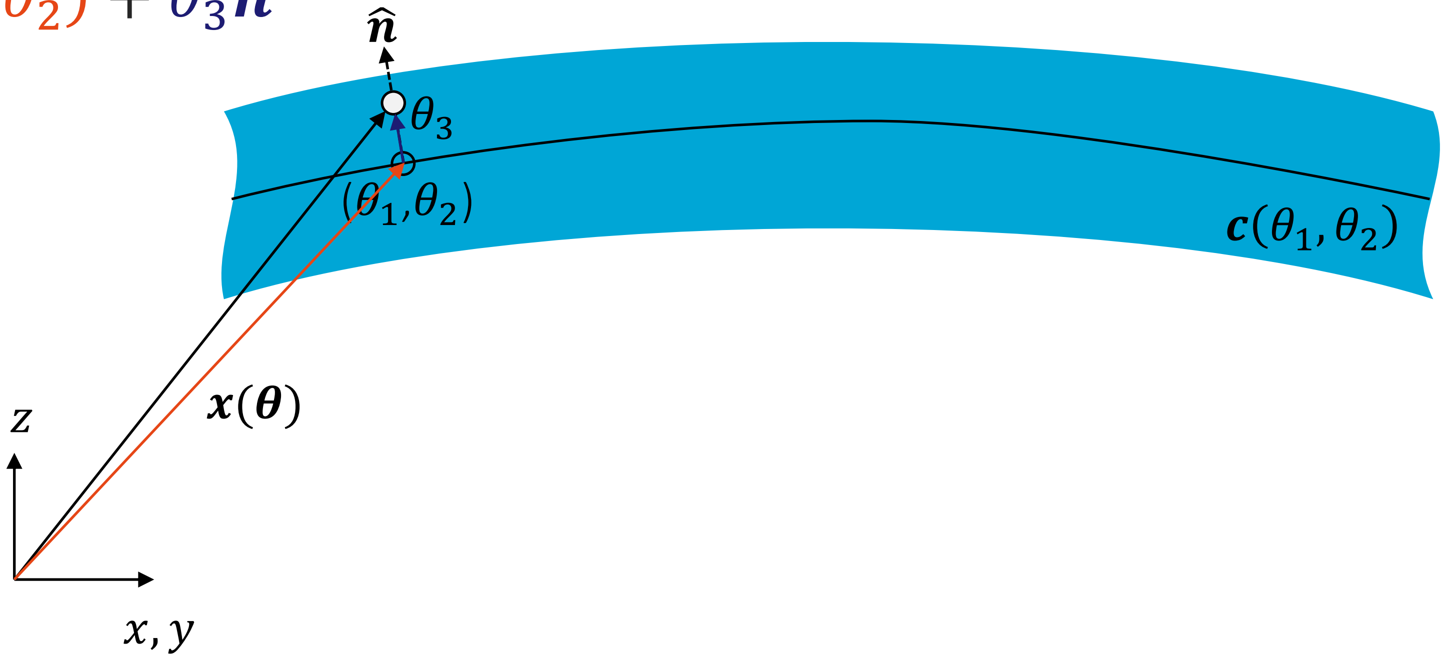


Kirchhoff-Love Shell
Coordinate System



Any point $\boldsymbol{x}$ on the (un)deformed shell can be represented by:

$$\boldsymbol{x}(\boldsymbol{\theta}) = \boldsymbol{c}(\theta_1, \theta_2) + \theta_3 \hat{\boldsymbol{n}}$$



Kirchhoff-Love Shell

Nonlinearities

Geometric Nonlinearity

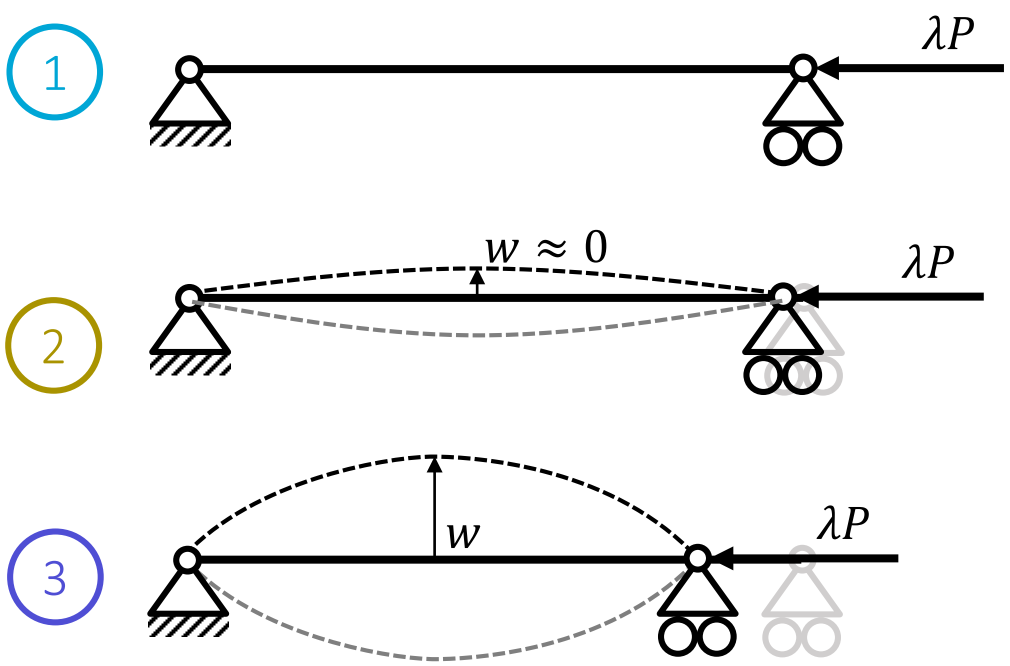
Material Nonlinearity

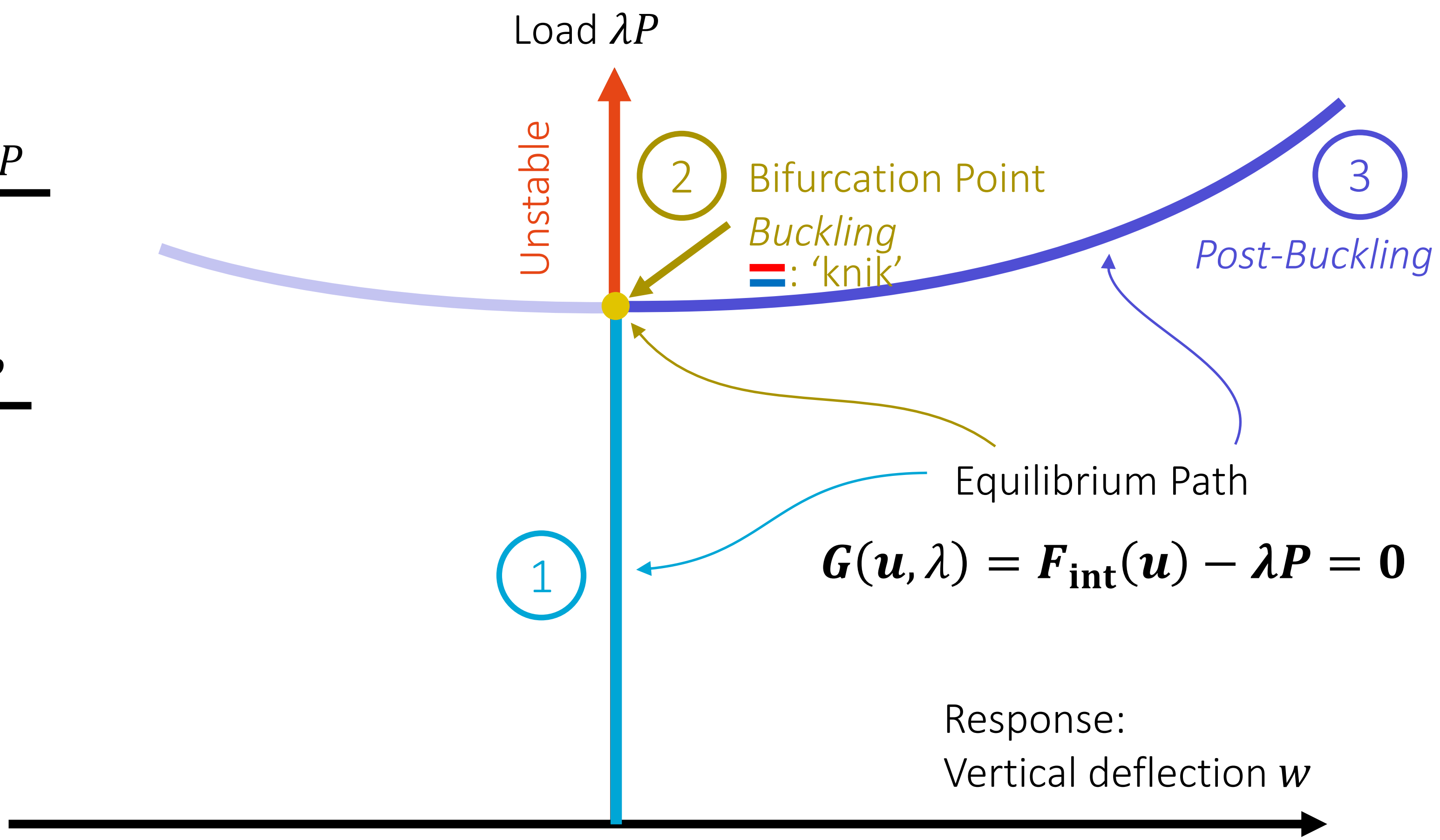
Methods

Extended Arc-Length Method

Extended Arc-Length Method

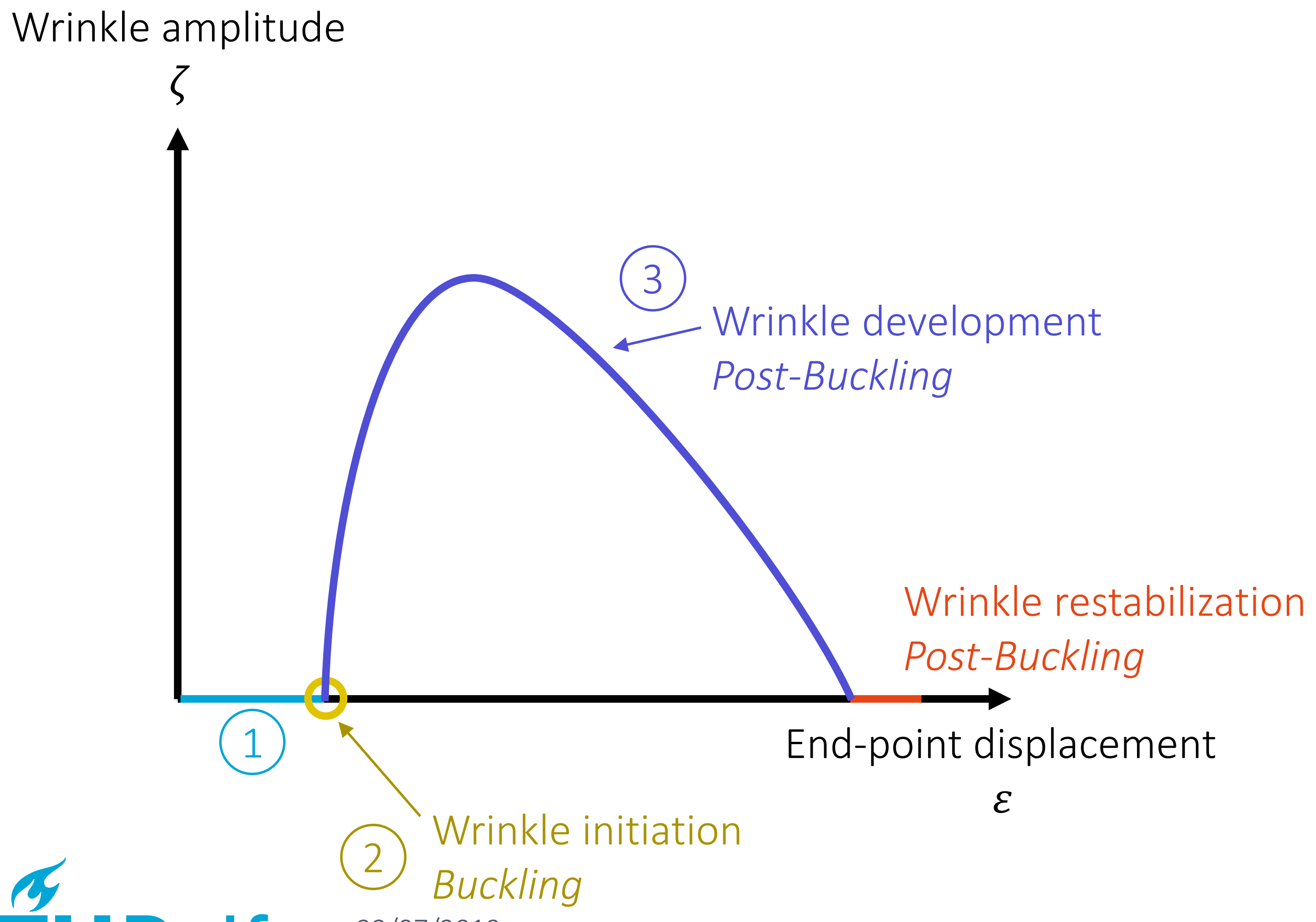
Buckling and Post-Buckling

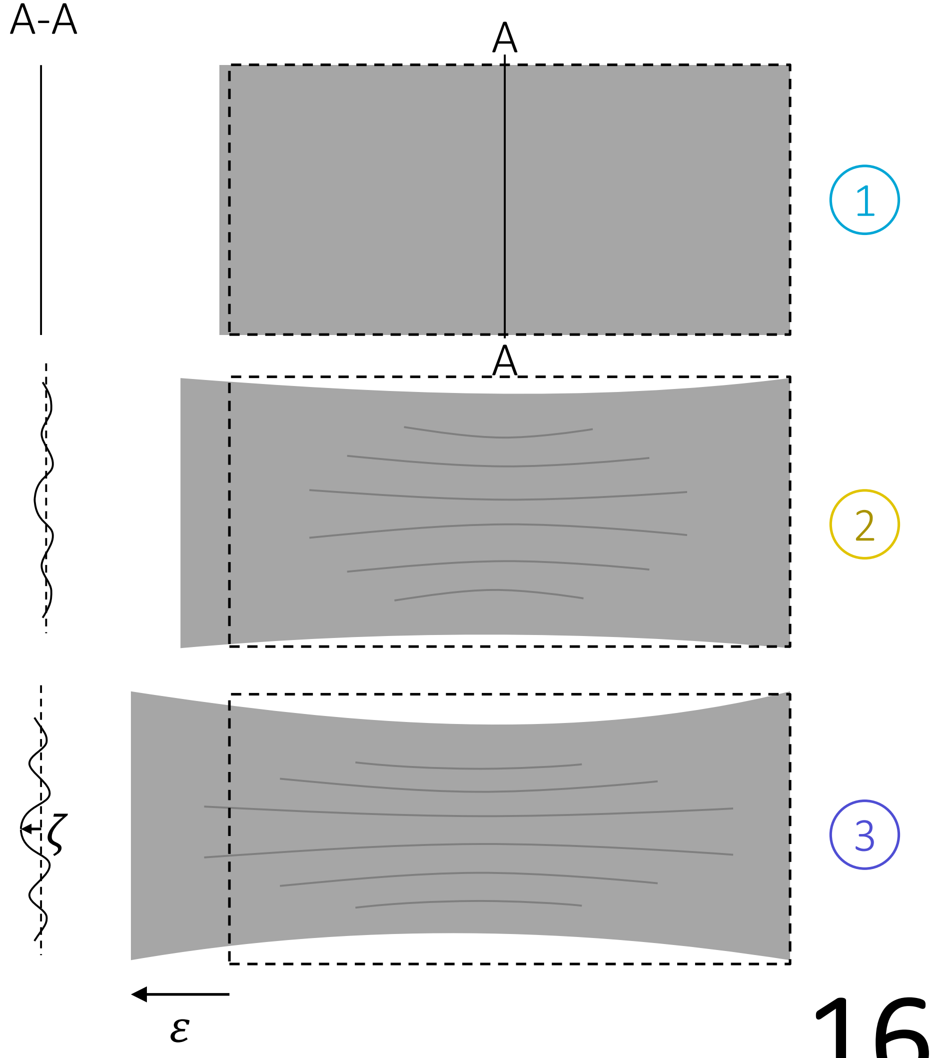




Extended Arc-Length Method

Wrinkling

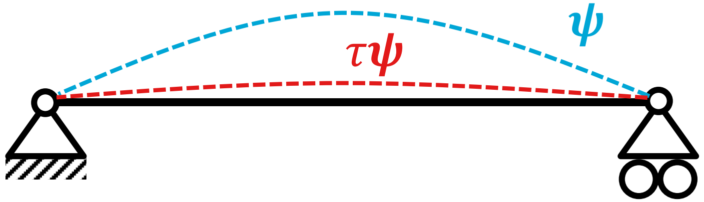




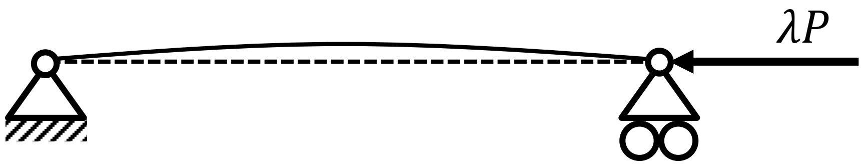
Extended Arc-Length Method

Ordinary Routine

1. Apply initial perturbation $\boldsymbol{\psi}$ with magnitude $\boldsymbol{\tau}$ to undeformed geometry



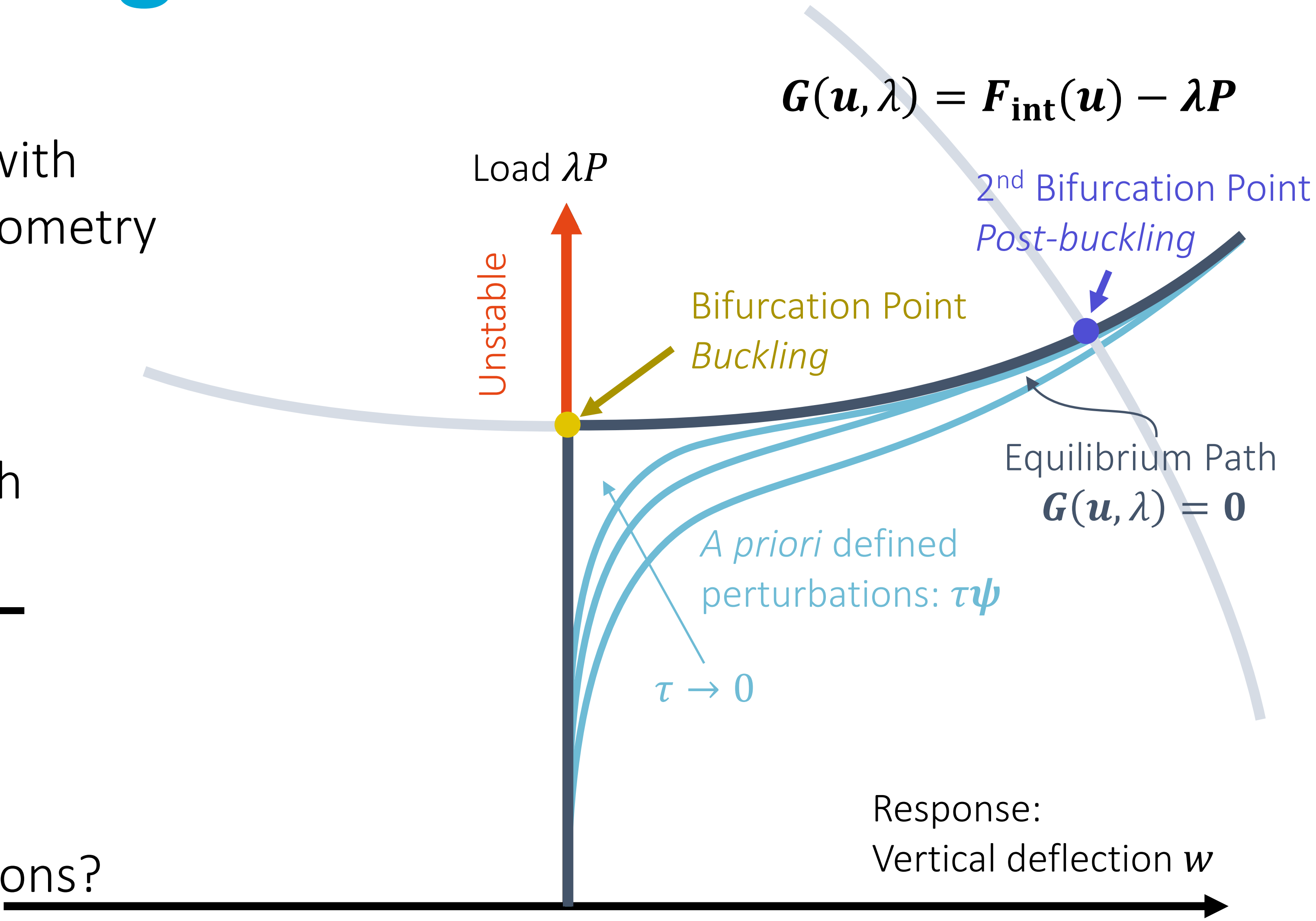
2. Compute the equilibrium path



Disadvantages

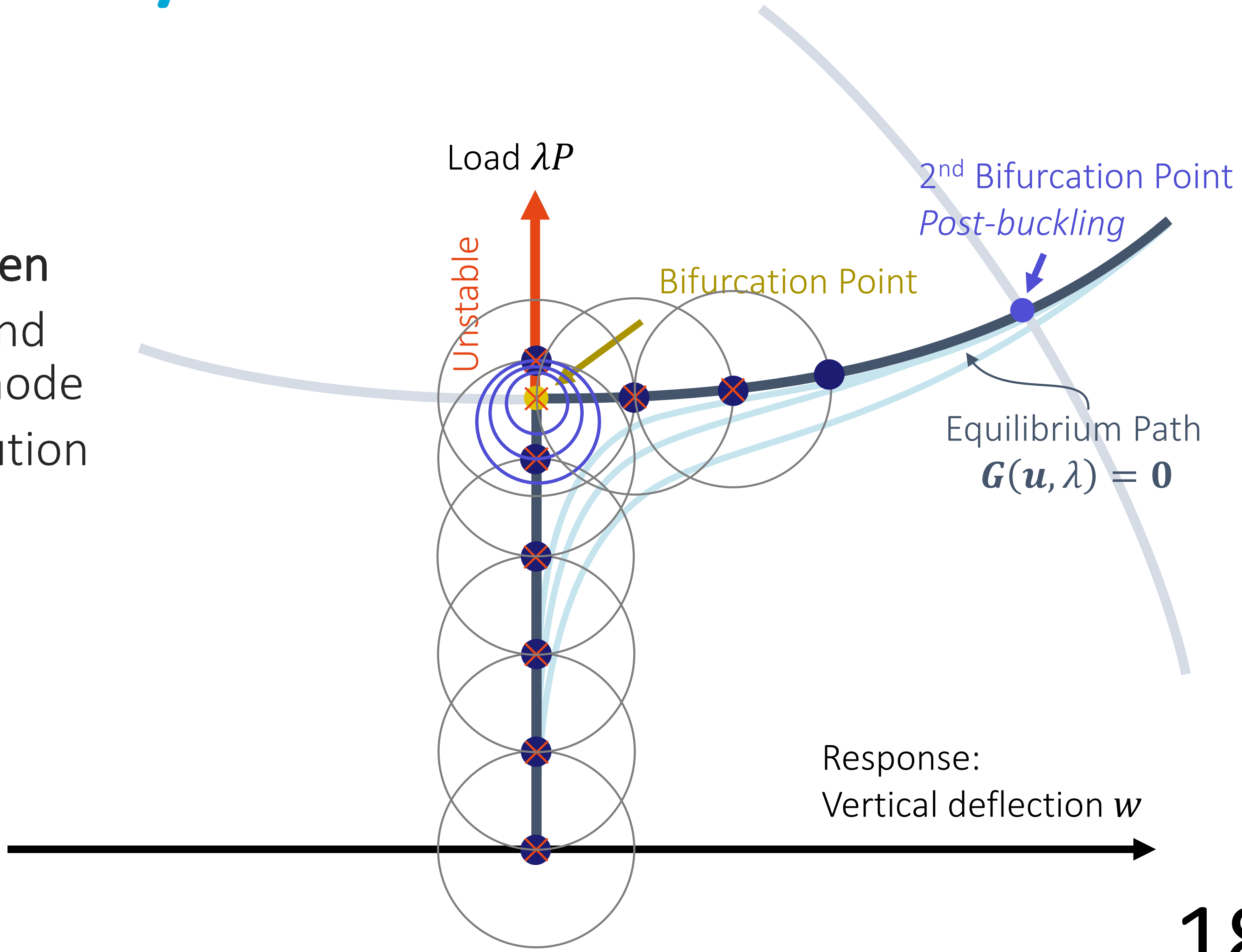
Choice of $\boldsymbol{\psi}$ and $\boldsymbol{\tau}$ subjective

What about secondary bifurcations?

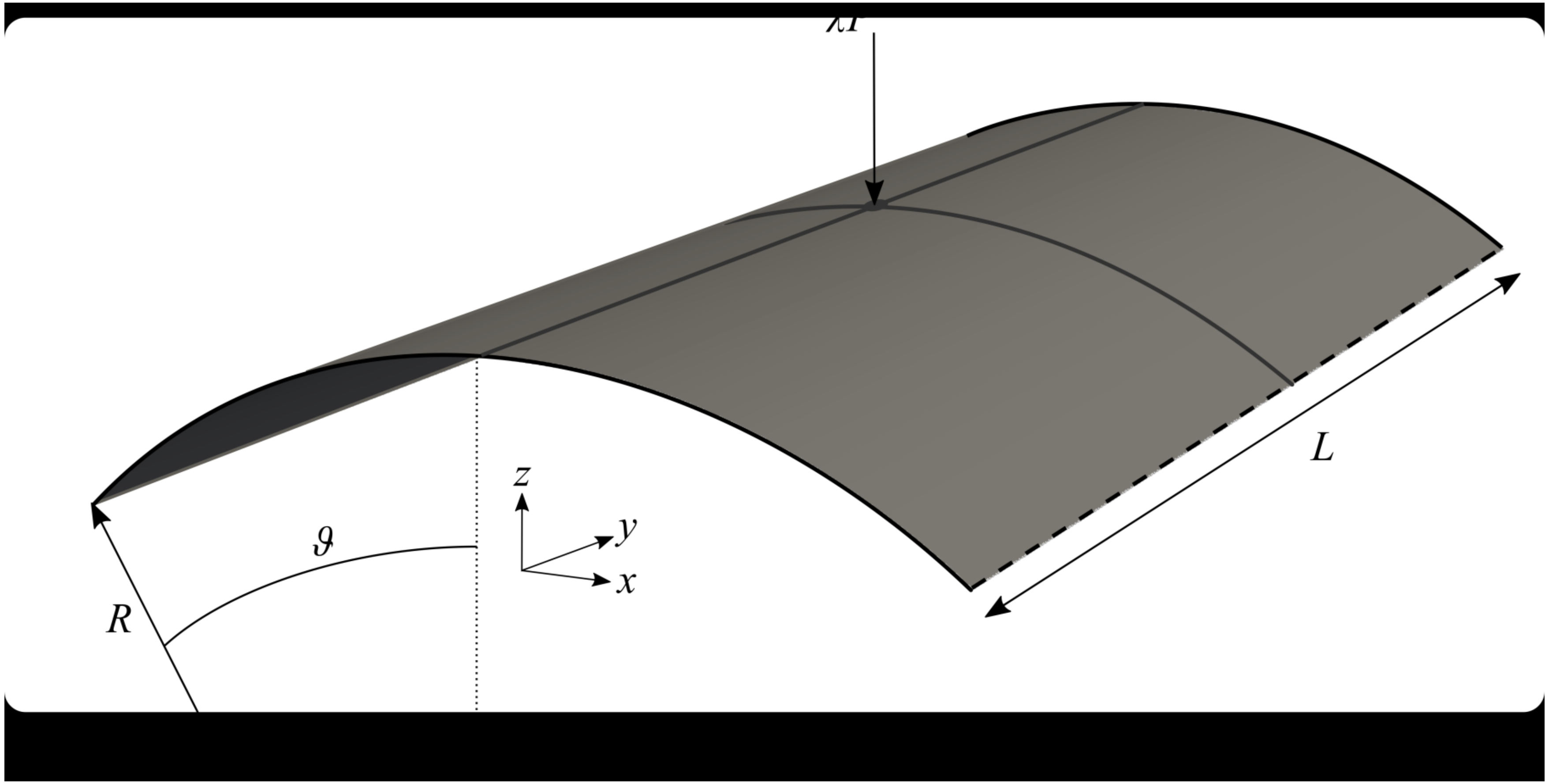


Post-Buckling Analysis
Present Algorithm

1. Follow equilibrium path
2. **If** unstable branch found **then**
   a. Compute *singular* point and corresponding buckling mode
   b. Apply deformation to solution
   c. Return to 1.



Extended Arc-Length Method
Path-Following



Extended Arc-Length Method

Path-Following

Load Control

Displacement Control

Arc-Length Control

Extended Arc-Length Method

Bifurcation Point Indication

Property of singular point:

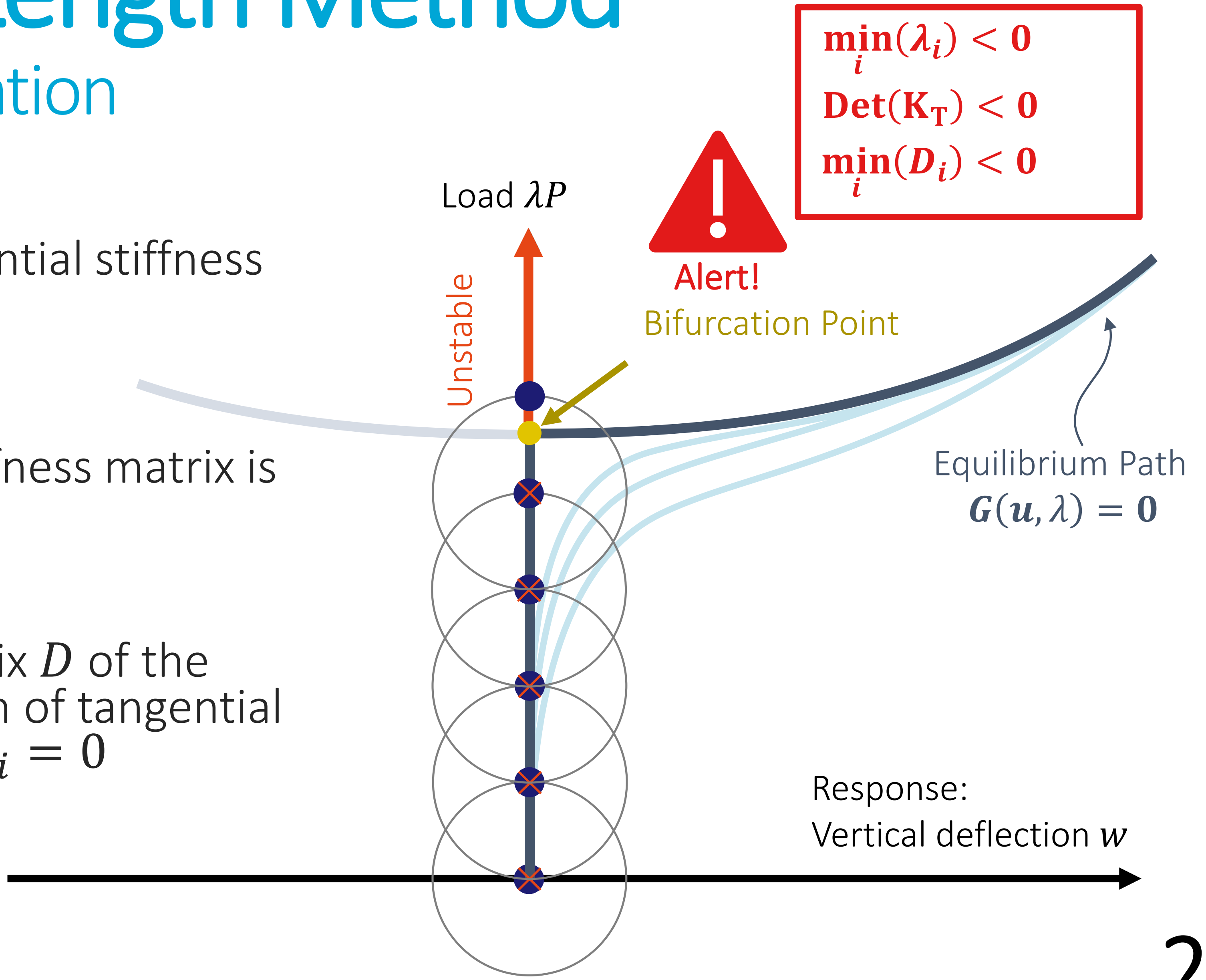
Minimum eigenvalue of tangential stiffness matrix is zero: $\min_i \lambda_i = 0$

Or

Determinant of tangential stiffness matrix is zero: $\mathrm{Det}\,(K_T) = 0$

Or

Lowest value of diagonal matrix D of the LDL^T Cholesky-decomposition of tangential stiffness matrix is zero: $\min_i D_i = 0$



Extended Arc-Length Method

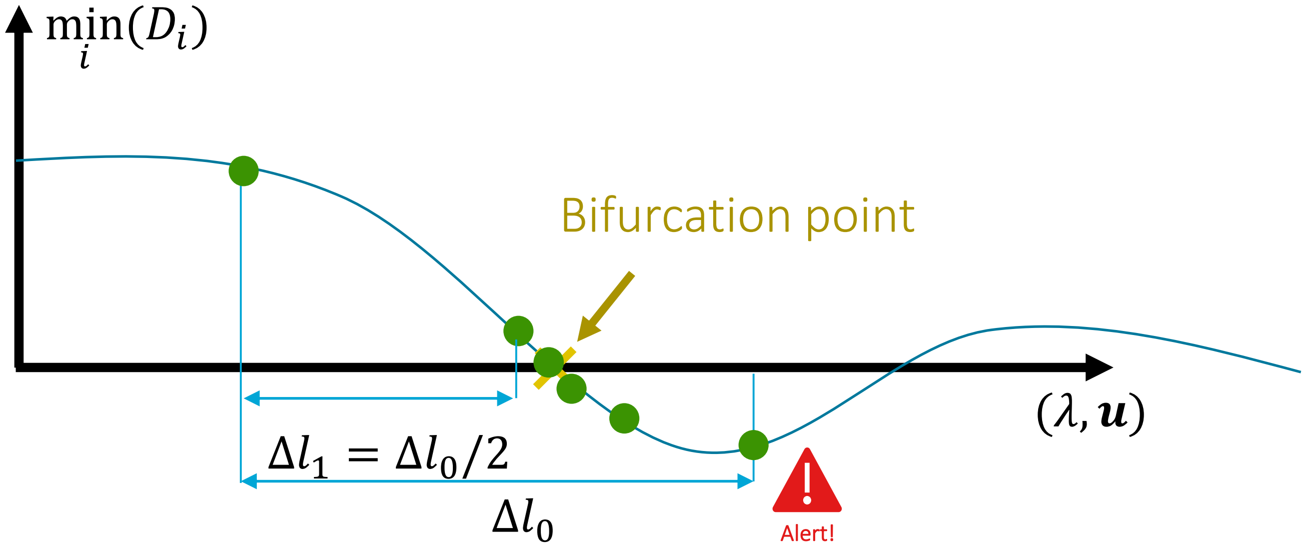
Bifurcation Point Approach

1. Bisection algorithm
   a. Robust
   b. Slowly converging (i.e. approaching)
2. Extended arc-length method, additional constraint equations:

$$K_T \boldsymbol{\psi} = \mathbf{0}$$
$$l(\boldsymbol{\psi}) = \|\boldsymbol{\psi}\| - 1 = 0$$

   a. Less robust
   b. Fast convergence

Until $|\min_i D_i| <$ tolerance



Extended Arc-Length Method

Branch Switching

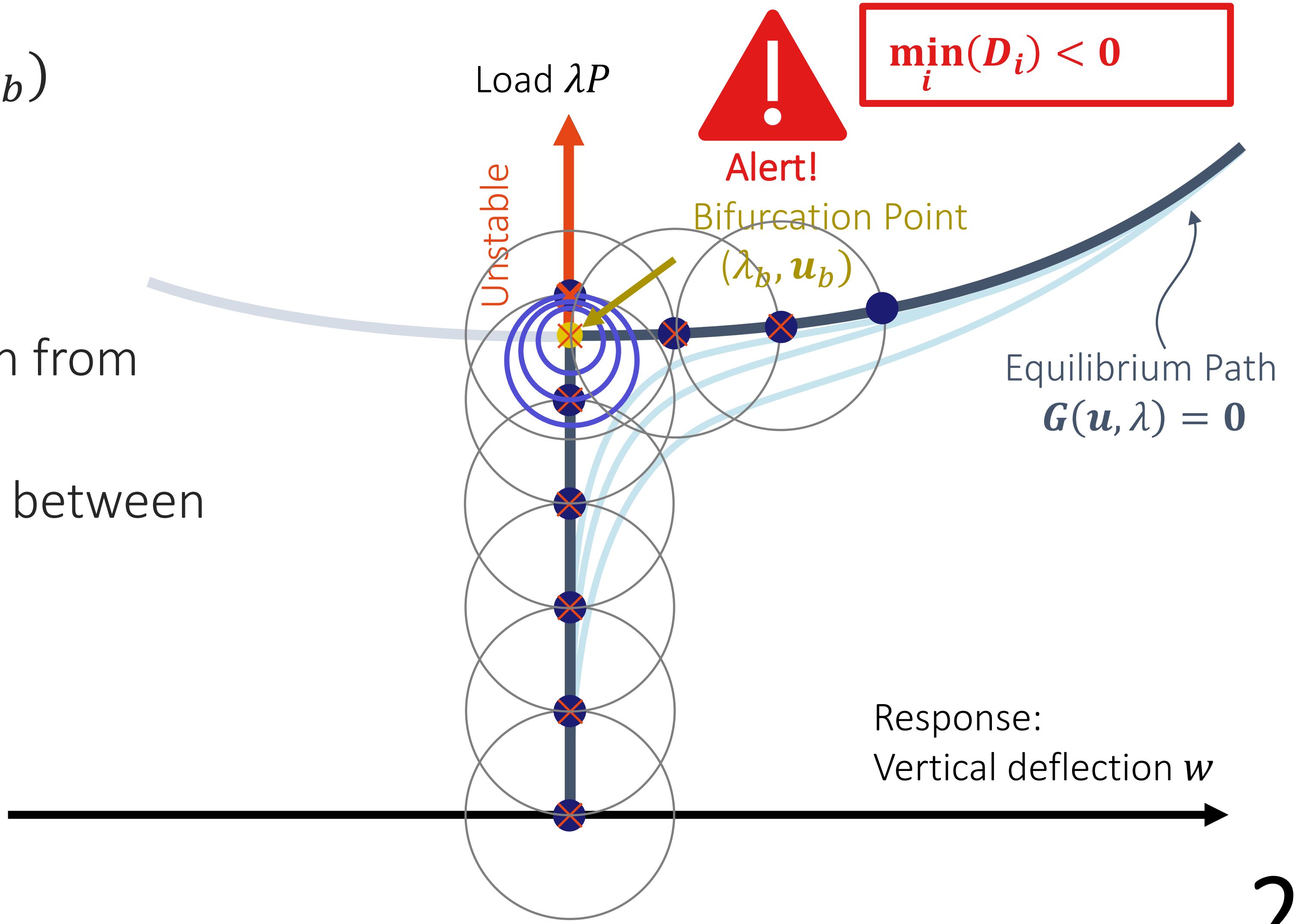
Bifurcation point found: $(\lambda_b, \boldsymbol{u}_b)$

Branch-switching by:

$$\boldsymbol{u} = \boldsymbol{u}_b + \tau\boldsymbol{\psi}$$
$$\lambda = \lambda_b$$

Buckling mode shape $\boldsymbol{\psi}$ known from extended method

Tuning magnitude τ; generally between 10^{-3} and 10^{-4}

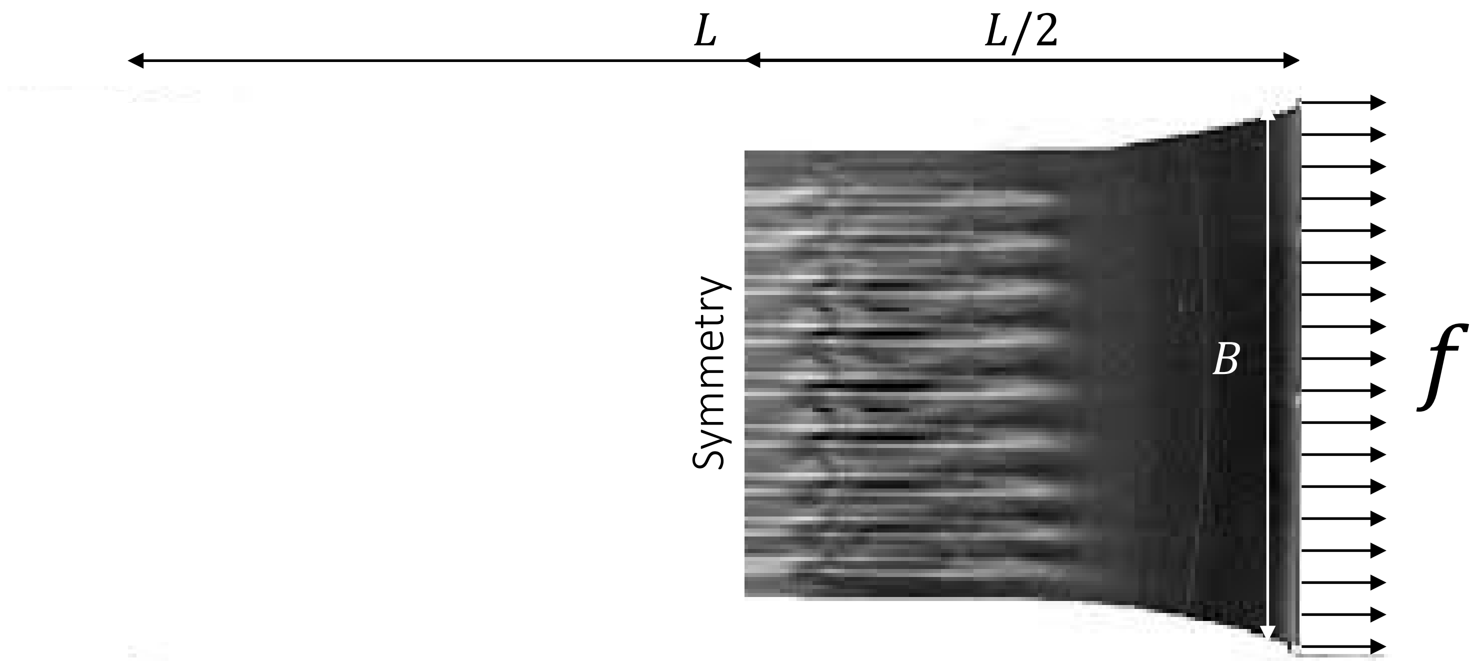


Results

Stretched Thin Sheet

TUDelft

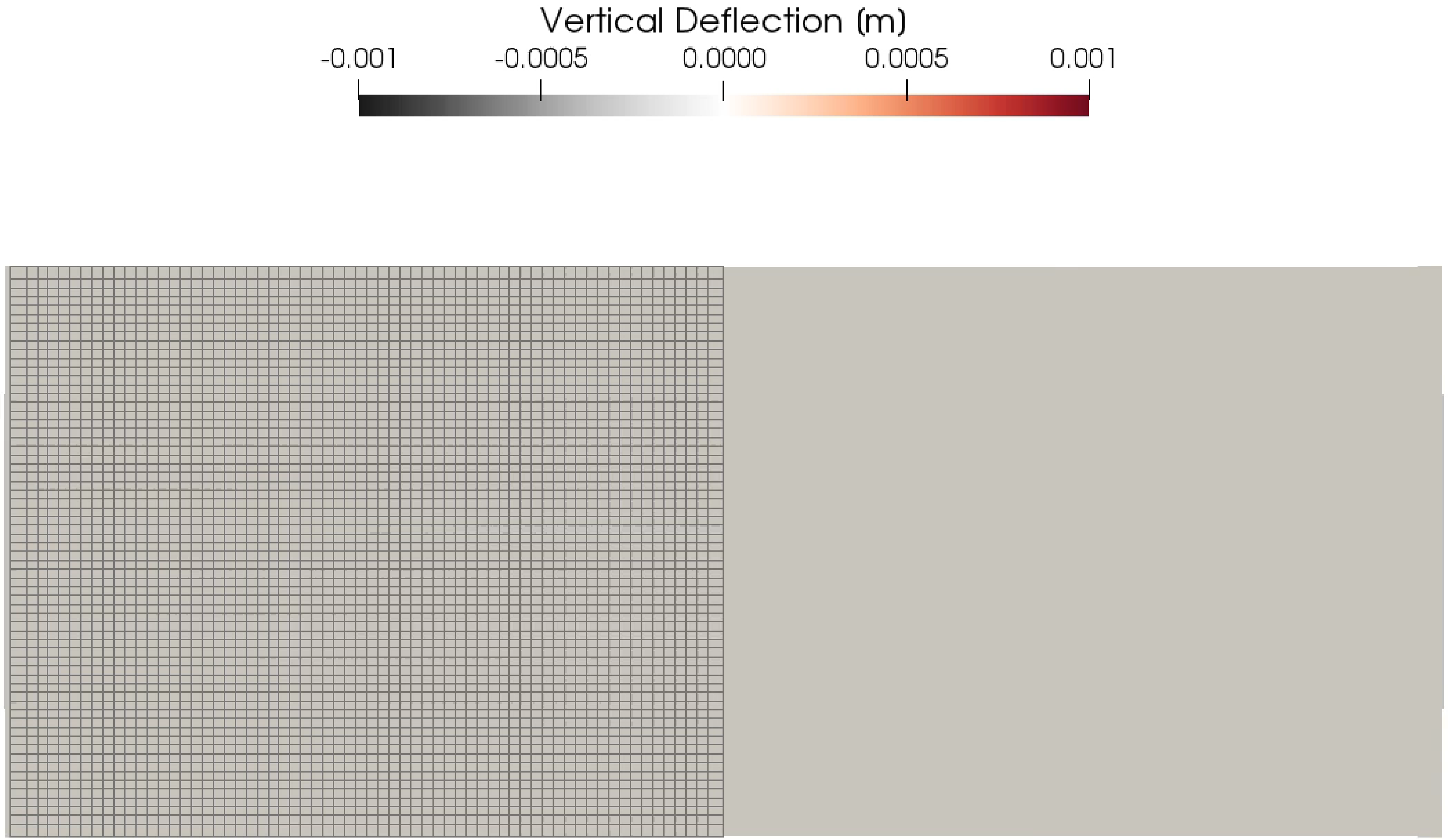
Stretched Thin Sheet
Model



Cerda *et al.* (2002), *Nature*

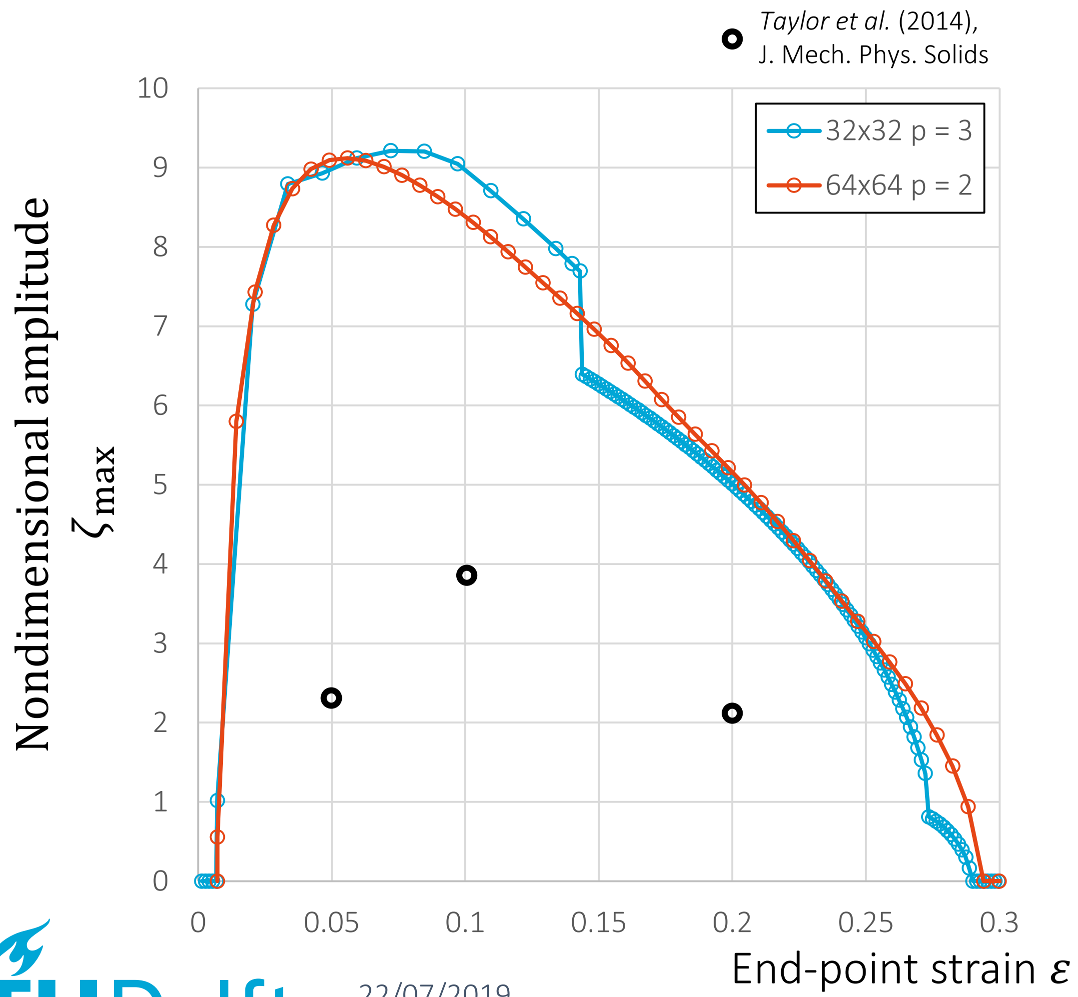
Stretched Thin Sheet

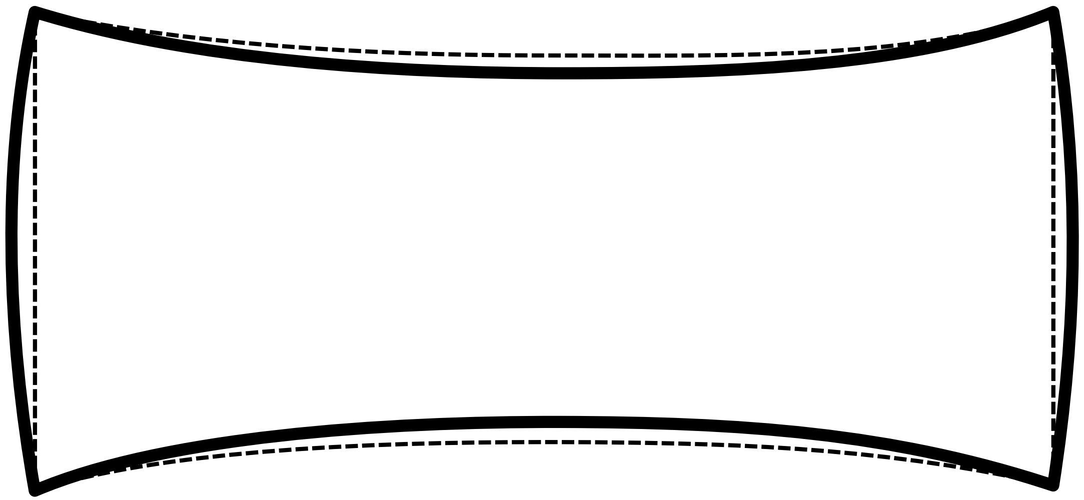
Results



Symmetric solution plotted for reference

Stretched Thin Sheet
Results





Conclusions & Recommendations

Conclusions

Research question: "How can *wrinkling formation* of large floating thin structures be numerically modelled with *Isogeometric Analysis*?"

Model perspective

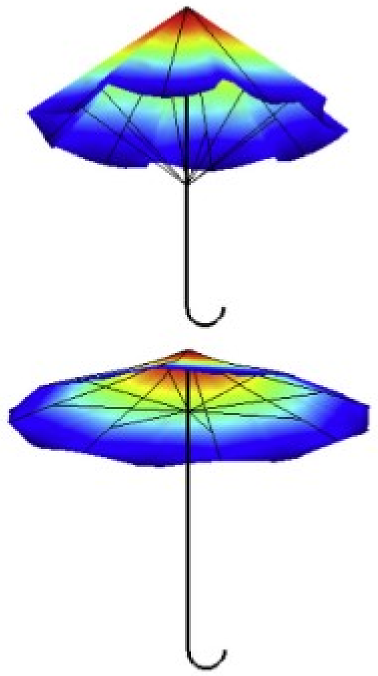
- Isogeometric Kirchhoff-Love shell model applicable to wrinkling of thin sheets.
- Present arc-length method provides post-buckling modelling technique without *a priori* user input
- Fine mesh needed for wrinkling prediction in stretched sheet

Recommendations

Research question: "How can *wrinkling formation* of floating thin structures be numerically modelled with *Isogeometric Analysis*?"

Model perspective

- Adaptive mesh refinement
- Straight edges of the stretched thin sheet
- Model applicability
  - Multi-patch coupling for more complex domains;
  - Periodic basis functions for circular domains
  - Element Coupling
  - Composite and rubber material models



Raknes et al. (2013),
Comput. Methods Appl. Mech. Eng.

Questions?