Solving instabilities at small timesteps in `CONTACT`

Interim literature presentation

Niels van der Wekken

Applied Mathematics, TU Delft

June 29, 2016

V$\mathcal{O}$Rtech

- Scientific Software Engineers
- 25 employees
- Delft, de Torenhove

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- Maintain, optimize software
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Distribution of `CONTACT`.

CONTACT

"Vollebregt & Kalker's rolling and sliding contact model"

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1978: DUVOROL
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1994: modernized
From here on many times extended and accelerated.

Wiggles

Time discretization Δt,
rolling velocity V,
space discretization Δx.

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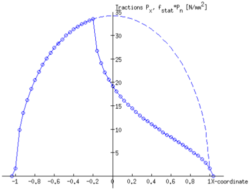
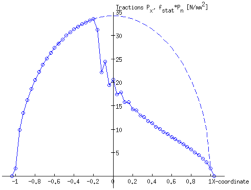
Instabilities arise when $c \ll 1$.

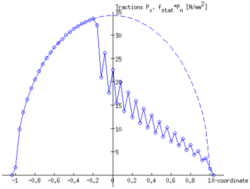
Wiggles



(a) Case: $c = 1$



(b) Case: $c = 0.1$



(c) Case: $c = 0.025$

Figure: The 2D Carter/Fromm problem using different timesteps.

Physics - deformation

Definition

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Particle in initial state has position $\mathbf{x}$.
Particle in deformed state has position $\mathbf{x} + \mathbf{u}$.

Physics - strain

Definition

Longitudinal strain: $\epsilon_{ii} = \frac{\partial u_i}{\partial x_i}$.

Definition

Shear strain: $\epsilon_{ij} = \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}, i \neq j$.

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Strain is relative displacement caused by stretching and bending of the body.

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Link strain to stress through the generalized Hooke's Law with elastic tensor C (a material property).

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Simplification in an isotropic material:

$$\sigma_{ii} = \lambda(\epsilon_{11} + \epsilon_{22} + \epsilon_{33}) + 2\mu\epsilon_{ii}$$

and

$$\sigma_{ij} = \mu\epsilon_{ij}, i \neq j$$

with material properties λ and μ.

Situation

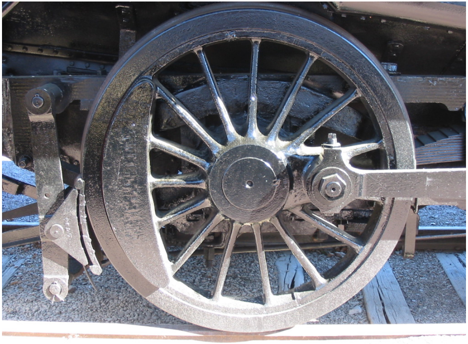

Figure: Train wheel on a rail.

Situation

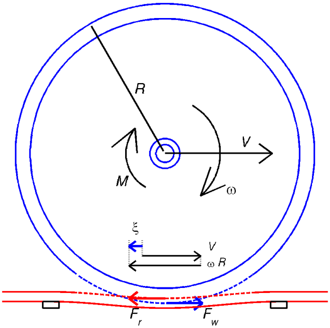

Figure: Schematic view of a wheel on a rail, where wheel and rail overlap there will be a contact area.

Traction

Surface stress of body 1: $\mathbf{p}^{(1)}$, of body 2: $\mathbf{p}^{(2)}$.
Newton's third law of motion:

$$\mathbf{p}^{(1)} = -\mathbf{p}^{(2)}.$$

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Split the normal and tangential surface stress into:

- the scalar normal stress p_n,
- the 2-vector tangential stress $\mathbf{p}_\tau$, the traction.

Displacement

After deformation a point on the surface of body 1 has moved from $\mathbf{x}^{(1)}$ to $\mathbf{x}^{(1)} + \mathbf{u}^{(1)}$ by an amount $\mathbf{u}^{(1)}$, this is called the displacement.

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Before deformation the normal distance between the bodies is:

$$h = \mathbf{x}_n^{(1)} + \mathbf{x}_n^{(2)}.$$

After deformation this normal difference becomes

$$e = h - \mathbf{u}_n.$$

Slip

Definition

Relative rigid slip $\mathbf{w} = \begin{bmatrix} \xi - \phi y \\ \eta + \phi x \end{bmatrix}$.

A function of the overall longitudinal creepage ξ, lateral creepage η, and spin creepage ψ.

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Definition

Relative slip $\mathbf{s} = \mathbf{w} + \frac{\dot{\mathbf{u}}}{V}$.

Both $\mathbf{s}$ and $\mathbf{w}$ are relative to the rolling velocity V.

Contact conditions

In the normal problem:

$$\text{in exterior area E}: \quad e > 0,\ p_n = 0$$
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$$\text{in adhesion area H}: \quad ||\mathbf{s}|| = 0,\ ||\mathbf{p}_\tau|| \leq g$$
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Coulombs friction law: $g = \mu p_n$.

Influence function

Link between the tractions and displacements:

$$u_i(\mathbf{x}) = \int_C A_{ij}(\mathbf{x}, \mathbf{y}) p_j(\mathbf{y}) dS$$

Boundary Element Methods

- Numerical approximation to solutions of PDE's.
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Advantages:

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Disadvantages:

\- When no Greens functions are known it is hard to find solutions in the interior.

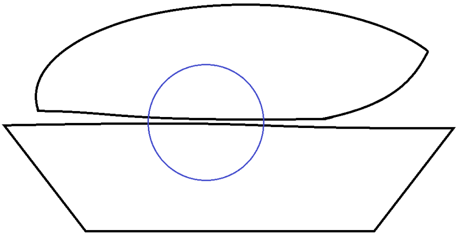
\- System of equations resulting from a BEM is usually dense.

Half Space approach

Contact area small compared to radius of curvature at contact.

"The contact stresses are highly concentrated close to the contact region and decrease rapidly in intensity with distance from the point of contact." - K.L. Johnson, *1985*

Approximate the real solution by the solution of the contact between two half spaces.



Quasi-identical behaviour

Simplification when the contacting bodies are geometrically and elasticlly symmetric.

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Geometric symmectic: half-space.

Elastic symmetry: for modulus of rigidity G and Poisson's ration ν we must have

$$\frac{1-2\nu_1}{G_1} = \frac{1-2\nu_2}{G_2}.$$

More simplifications

- Going from full 3D to 2D.
- Transient rolling vs steady-state.

Discretization

Slip equations: relations between slip and displacements.
Contact conditions: relations between slip and tractions.

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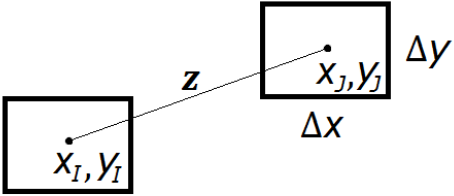

$$A_{IiJj} = \int_{x_J - x_I - \frac{\Delta x}{2}}^{x_J - x_I + \frac{\Delta x}{2}} \int_{y_J - y_I - \frac{\Delta y}{2}}^{y_J - y_I + \frac{\Delta y}{2}} A_{ij}(\mathbf{z}) dz_2 dz_1$$

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In discretization a factor $A(\mathbf{x}) - A'(\mathbf{x} + \mathbf{dq})$ arises.

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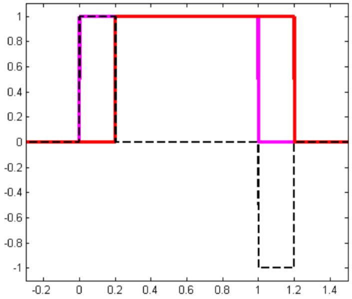
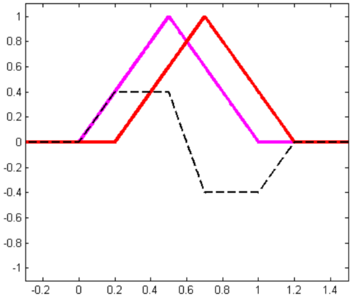
Possible cause? Piecewise-approximation.





Research Questions

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2. Does replacing the piecewise constant basis functions by piecewise (bi)linear basis functions solve this problem?
   - If this is not the case, how can we solve it?
3. How does replacing the piecewise constant basis functions by piecewise (bi)linear basis functions influence the rate of convergence of the algorithm?

Thank you!

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Questions?