# Final Master Thesis Project proposal: Isogeometric Analysis for Compressible Flows with Application in Turbomachinery

## Introduction

Since its introduction in 2005, *isogeometric analysis* (IgA) has experienced a lot of interest in the scientific community both from practitioners as well as from theoretical analysts. The key idea in isogeometric analysis is to directly integrate finite element analysis (FEA) into the computer aided design (CAD) tool by using a common geometry and data representation. That is, the same set of basis functions, e.g., (hierarchical) B-splines or more often non-uniform rational B-splines (NURBS), are adopted to describe the computational geometry in the CAD tool *and* to span the solution space for FEA. As a result, the discretisation does not introduce an additional approximation error at the boundary due to an inexact representations of curved surfaces which is typically the case in low-order and/or sub-parametric finite elements. Isogeometric analysis gained quite some popularity in the fields of (incompressible) fluid-structure interaction and structural shape optimisation, where the ability to represent the geometry exactly can be immediately turned into higher solution accuracies.

At first glance, the global higher continuity that this approach offers over classical finite elements does not seem to be of great help when applying isogeometric analysis to compressible flows, such as those modelled by the compressible Euler and Navier-Stokes equations, which may exhibit discontinuities, shock waves and steep gradients. In fact, the use of IgA in hydrodynamics is rather uncommon although recent publications demonstrate the general applicability of IgA to hydrodynamics [Bas13,Bas14,Tro12].

## Problem description

This master thesis project will focus on the realisation of an inviscid compressible flow solver for a single prototypical turbine blade modelled in the framework of isogeometric analysis. It is well known, that compressible flows may give rise to solutions with steep gradients and discontinuities so that the use of stabilised discretisation techniques such are mandatory to prevent the generation of spurious oscillations and unphysical solutions. In this project, the algebraic flux correction (AFC) methodology, that was originally developed in the context of low-order continuous Galerkin finite element methods shall be generalised to IgA based on higher-order B-splines. B-spline basis functions feature some nice properties which make them a natural candidate for the generalisation of AFC techniques: positivity of basis functions over their entire support, partition of unity property, and the fact that low-order B-splines coincide with low-order Lagrange finite elements which have been shown to work in the AFC context.

A second part of the thesis work will be concerned with the calculation of flow induced forces acting on the turbine blade and - if time permits - the resulting deformation of the solid. The turbine blade will be considered fixed and described by a linear elasticity model, which suffices to illustrate a prototypical fluid-structure interaction.

The implementation should be carried out in one of the available open-source packages, e.g., PetIGA, GeoPDEs, Igatools, Igafem, which provide the basic IgA framework but lack the AFC methodology and/or an inviscid compressible flow solver. However, most of the packages already contain an implementation of linear elasticity.

## Challenge

The main challenge of this master project is to generalise the AFC methodology to high-order B-spline basis functions and analyse the numerical properties of the resulting discretisation. Pre-studies indicate that the straightforward generalisation is possible but additional care must be taken to prevent peak-clipping of smooth local extrema. The aim for this thesis work is to consider an alternative strategy which makes use of the fact that higher-order B-spline functions are given by a combination of B-splines of one order less. This recursive definition suggests the use of a hierarchical limiting approach. All in all, being a topic of active research makes this topic very challenging.

## Time schedule

The following tasks are foreseen:

- Literature study in order to understand the concept of isogeometric and finite element analysis, to get background information about the state-of-the-art in numerical solution techniques for compressible flows and linear elasticity, and to get an overview of available open-source IgA software packages;
- Start with a prototypical IgA implementation of the one-dimensional convection-diffusion problem in MATLAB using a standard B-spline basis; investigate the performance of stabilisation and/or flux limiting techniques for higher-order approximations in the presence of dominating convection;
- Realise/implement a flow solver for the compressible Euler equations in one of the existing open-source IgA packages;
- Investigate the flow solver for a single prototypical turbine blade (assuming periodic boundary conditions and/or cylindrical coordinates);
- If time permits, calculate the forces acting on the turbine blade and realise a prototypical one-way FSI-coupling using a linear elasticity model for the blade.

## Contact

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## Literature

[Bas13] Y. Bazilevs, I. Akkerman, D.J. Benson, G. Scovazzi, M.J. Shashkov, Isogeometric analysis of Lagrangian hydrodynamics, Journal of Computational Physics, 243, 2013, p. 224-243, DOI: 10.1016/j.jcp.2013.02.021.

[Bas14] Y. Bazilevs, C.C. Long, I. Akkerman, D.J. Benson, M.J. Shashkov, Isogeometric analysis of Lagrangian hydrodynamics: Axisymmetric formulation in the rz-cylindrical coordinates, Journal of Computational Physics, 262, 2014, p. 244-261, DOI: 10.1016/j.jcp.2014.01.001.

[Cot09] J.A. Cottrell, T.J.R. Hughes, Y. Bazilevs, Isogeometric Analysis: Towards Integration of CAD and FEA. Wiley and Sons, 2009.

[Tro12] P. Trontin, Isogeometric analysis of Euler compressible flow. Application to aerodynamics, AIAA 2012-0295, DOI: 10.2514/6.2012-295.