

# MSc-project: High-Order Material Point Method

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Joint MSc-project by



## Motivation

The *Finite Element Method* (FEM) is a well-established tool for the computational study of all types of engineering problems. However, when it comes to the analysis of problems involving large deformations of solids, the standard FEM has its limitations - one may think of simulations of slope failures or the impact of a bullet on tissue. Severe distortions of the finite element mesh that is used for the discretization of the solid give rise to numerical inaccuracies and can even lead to tangled meshes. Alternative numerical methods such as the *Material Point Method* (MPM) have been developed to overcome this drawback [Su94, Su95]. The MPM makes use of a fixed background FE mesh to assemble and solve the considered equilibrium equations of a time step and a cloud of material points moving through the mesh accounting for the arbitrarily deforming solid.

Within MPM, linear functions are commonly used for the interpolation of nodal quantities on the background FE mesh, whereas high-order interpolation functions would be desirable. Firstly, high-order functions enable a more accurate reproduction of physical quantities such as stresses. Moreover, interpolation of higher order makes it possible to utilize coarser FE meshes, thus reducing the computational costs while preserving the same spatial accuracy or even improving it. However, with MPM, the use of standard quadratic finite element basis functions (i.e. based on Lagrange polynomials) leads to non-physical effects [An10] not encountered with finite elements using linear interpolation functions.

Extension of the MPM to higher-order B-Spline interpolation functions as employed in the Isogeometric Analysis framework [Hu05] is expected to eliminate these non-physical effects, thus enabling more accurate and faster numerical analyses. Furthermore, the use of quadratic B-Spline interpolation functions is expected to solve numerical problems found with MPM that are related to the discontinuity of interpolation function derivatives across element boundaries with standard  $C^0$ -continuous finite elements [St08].

At Deltares, a Dutch research and consulting company situated in Delft, a 3D MPM is being developed for the study of dynamic geomechanical problems in the frame of a research community consisting further of University of Cambridge, UPC Barcelona, TU Hamburg-Harburg, University of Padova and TU Delft. In the frame of a current joint STW research project of TU Delft, University of Utrecht and Deltares, underwater slope failures will be investigated with the MPM. Long computing times are expected in view of the large spatial extent of such slides stretching over tens or hundreds of meters. A reduction of the computational costs is of high relevance to perform such analyses in reasonable time.

## Problem description and challenges

The task of this master thesis is to develop a high-order Material Point Method that makes use of quadratic B-Spline basis functions for the accurate interpolation of quantities on unstructured FE background triangulations. This requires combining ideas from isogeometric analysis with the MPM. The main challenges are the design of quadratic (or even cubic) B-Spline basis functions with higher-order inter-element continuity on unstructured triangulations and their integration into the element-by-element framework and data structures underlying standard finite element codes.

## Time schedule

The following tasks are foreseen:

The student will first familiarize with the FEM and MPM, both being closely related. Here, besides a literature study on MPM, a 1D FEM/MPM will be implemented in Matlab using linear interpolation functions.

Afterwards, the 1D FEM/MPM Matlab implementation will be extended to quadratic interpolation functions. The use of standard high-order finite elements with MPM is known to entail problems of numerical stability [Be12], which will require special attention in this study. In particular, both standard quadratic interpolation functions and higher-order B-Splines will be implemented and their properties (accuracy, stability, etc.) will be analyzed for simple benchmark problems that possess analytical solutions.

In a next step, the 1D FEM/MPM Matlab implementation will be extended to two dimensions, whereby a triangulation of the domain is adopted as FE background mesh. The extension of B-Spline interpolation functions to triangles with higher-order inter-element continuity represents a challenging task [Ja14, Sp12]. The student is expected to perform a literature study on existing approaches and decide on one of them that will be implemented into the 2D FEM/MPM Matlab code. The numerical properties of the novel B-spline enhanced MPM will be studied numerically for well-established benchmark problems.

The master thesis should conclude with recommendations on how to integrate the concept of higher-order B-Spline interpolation functions elaborated on in this thesis into the MPM code developed by Deltares and partners, e.g. by making use of Bezier extraction techniques. This MSc-project is closely related to an ongoing PhD study jointly supervised by Deltares and TU Delft, Numerical Analysis group.

## Contact

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## Literature

- [An10] S. Andersen, L. Andersen. Analysis of spatial interpolation in the material-point method, *Computers and Structures*, 88, pp. 506-518, 2010.
- [Be12] L. Beuth. Formulation and Application of a Quasi-Static Material Point Method. PhD thesis, Institute of Geotechnical Engineering, University of Stuttgart, 2012.

- [Hu05] T.J.R. Hughes, J.A. Cottrell, and Y. Bazilevs. Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. *Computer Methods in Applied Mechanics and Engineering* 194, pp. 4135-4195, 2005.
- [Ja14] N. Jaxon, X. Qian. Isogeometric analysis on triangulations. *Computer-Aided Design* 46, pp. 45-57, 2014.
- [Sp12] H. Speleers, C. Manni, F. Pelosi, and M.L. Sampoli. Isogeometric analysis with Powell-Sabin splines for advection-diffusion-reaction problems. *Computer Methods in Applied Mechanics and Engineering* 221-222, pp. 132-148, 2012.
- [St08] M. Steffen, R.M. Kirby, and M. Berzins. Analysis and Reduction of Quadrature Errors in the Material Point Method (MPM). *International Journal for Numerical Methods in Engineering* 76 (6), pp.022-948, 2008.
- [Su94] D. Sulsky, Z. Chen, and H.L. Schreyer: A particle method for history-dependent materials. *Computer Methods in Applied Mechanics and Engineering* 118, pp. 179- 196, 1994.
- [Su95] D. Sulsky, S. Zhou, and H.L. Schreyer. Application of a particle-in-cell method to solid mechanics. *Computer Physics Communications* 87, pp. 236-252, 1995.